RCE AND LABOR

AND GEODETIC SURVEY

O. H. TITTMANN, SUPERINTENDENT

DIRECTIONS

FOR

MAGNETIC MEASUREMENTS

BY

DANIEL L. HAZARD

COMPUTER, DIVISION OF TERRESTRIAL MAGNETISM

WASHINGTON
GOVERNMENT PRINTING OFFICE
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DEPARTMENT OF COMMERCE AND LABOR

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PREFACE.

Although the principles involved in the measurements of the earth's magnetism have not changed since the publication of the third edition of "Directions for Magnetic Observations with Portable Instruments," * the methods of observing and the instruments used have received so many modifications as the result of accumulated experience that a new presentation of the subject has been needed for some time to facilitate the field work and to secure uniformity. In addition it is important that the principles involved should be explained in more detail than was done in the above-mentioned publication, so that the observer may have a better understanding of what he is doing and why he is doing it without being obliged to refer to other publications. Moreover, with the establishment of five magnetic observatories and the inauguration of magnetic observations on board the vessels of the Survey, the need has arisen for printed directions for making the observations required in those two branches of the magnetic work of the Survey. The endeavor will be made to present the subject-matter in such form that an observer familiar with the use of instruments of precision but without experience in magnetic work may be able to make in a satisfactory manner the various observations incident to the determination of the magnetic elements without other assistance than that to be obtained from these directions.

In the preparation of this paper the following publications have been consulted:

Principal Facts of the Earth's Magnetism, by L. A. Bauer. Washington, Government Printing Office, 1909. (Reprinted from U. S. Magnetic Declination Tables, 1902.)

Theory of Magnetic Measurements, by F. E. Nipher. New York, 1886.

Spherical and Practical Astronomy, by Wm. Chauvenet. Philadelphia, 1887.

Traité de Magnétisme Terrestre, by E. Mascart. Paris, 1900.

Erdmagnetismus, Erdstrom und Polarlicht, by Dr. A. Nippoldt, jr. Leipzig, 1903.

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*Appendix No. 8, C. & G. S. Report for 1881. Gov't Printing Office, 1882.

Handbuch des Erdmagnetismus, by J. Lamont. Berlin, 1849.

Ableitung des Ausdrucks für die Ablenkung eines Magnetnadel durch einen Magnet, by Dr. Börgen. Hamburg, 1891.

Collimator Magnets and the Determination of the Earth's Horizontal Force, by Charles Chree. Proceedings Roy. Soc. London, No. 419, 1899.

The Law of Action between Magnets, by Charles Chree. London, Edinburgh, and Dublin, Phil. Magazine, August, 1904.

La Section Magnétique de l'observatoire de l'Èbre, by E. Merveille, S. J. Barcelone, 1908.

Elementary Practical Physics, by Stewart and Gee. London, 1887.

Elements of the Mathematical Theory of Electricity and Magnetism, by J. J. Thomson, Cambridge, England, 1897.

A Treatise on Magnetism and Electricity, by Andrew Gray. London, 1898.

A Physical Treatise on Electricity and Magnetism, by J. E. H. Gordon. New York, 1880.

Practical Problems and the Compensation of the Compass, by Diehl and Southerland. Washington, Government Printing Office, 1898.

Admiralty Manual for the Deviation of the Compass, by Evans and Smith. London, 1901.

The subject will be treated under the following general headings:

I. Theory of magnetic measurements, including some of the more important facts about the earth's magnetism and the methods employed for determining instrumental constants.

II. Directions for absolute observations on land.

III. Directions for observations at sea.

IV. Directions for operating a magnetic observatory.

THEORY OF MAGNETIC MEASUREMENTS.

THE EARTH'S MAGNETISM.

INTRODUCTION.

Whether the earth is a great magnet or simply acts as a magnet as the result of electric currents flowing about it, it is surrounded by a magnetic field, and the measurements of the earth's magnetism at any place consist in determining the direction and intensity of that field.

A magnet suspended in such a way as to be free to turn about its center of gravity would take a position with its magnetic axis tangent to the lines of force of the earth's magnetic field. As it is practically impossible to suspend a magnet in that way, it is usual to determine the direction of the earth's magnetic field by means of two magnets, one constrained to rotate in a horizontal plane and the other in a vertical plane.

MAGNETIC ELEMENTS.

The magnetic meridian at any place is the vertical plane defined by the direction of the lines of force at that place.

The magnetic declination, D, is the angle between the astronomic meridian and the magnetic meridian and is considered East or West according as the magnetic meridian is east or west of true North. Declination is often called *variation of the compass* or simply *variation*.

The *dip* or *inclination*, I, is the angle which the lines of force make with the horizontal plane.

Instead of measuring the *total intensity*, F, of the earth's magnetic field, it is usually more convenient to measure its *horizontal component*, H. These three quantities, *declination*, *dip*, and *horizontal intensity*, are usually spoken of as the *magnetic elements* and from them the total intensity and its components in the three coordinate planes may be computed by means of the simple formulas:

$$F = H \sec I \qquad Y = H \sin D$$
$$X = H \cos D \qquad Z = H \tan I$$

X and Y being the components in the horizontal plane, X directed north (+) or south (−) and Y directed east (+) or west (−), and Z being the component directed vertically downward.

UNITS OF MEASURE OF INTENSITY.

The intensity of a magnetic field is the force which a unit pole would experience when placed in it. A unit pole is one which repels an equal pole at unit distance with unit force.

At the present time almost all measurements of the intensity of the earth's magnetic field are made in terms of the C. G. S. system, in which the fundamental units are the centimeter, the gram, and the second. Before the metric system came into general use, it was customary in English-speaking countries to use the British system of units, based on the foot, the grain, and the second. To convert measures of intensity expressed in British units into their equivalents in the C. G. S. system, they must be multiplied by the factor 0.046108 (logarithm $= \bar{8}.66378$).

DISTRIBUTION OF THE EARTH'S MAGNETISM.

The *magnetic poles* of the earth are those points on its surface at which the dip needle stands vertical and toward which the compass needle points, throughout the adjacent region. The north magnetic pole is approximately in latitude 70° N. and longitude 97° W., and the south magnetic pole in latitude 73° S. and longitude 156° E. It must be borne in mind that these *magnetic poles* have not the characteristics of the poles of a bar magnet. If they had, there should be an enormous increase in the total intensity when approaching the poles, which is not the case. They are not even the points of maximum intensity, there being four areas, two in each hemisphere, in which the total intensity is greater. The earth acts like a great spherical magnet; that is, a bar magnet at its center which would produce the effects observed at the surface would have its poles practically coincident.

If the earth were uniformly magnetized, its magnetic poles would be at the opposite extremities of a diameter, the magnetic meridians would be arcs of great circles, and a comparatively small number of observations would suffice to determine the distribution of magnetism over its surface. As a matter of fact, according to Bauer, only about two-thirds of the earth's magnetism can be represented by a uniform magnetization and the distribution of the remainder is very irregular, representing the resultant effect of irregularities which are continental, regional, or purely local in extent. These local irregularities or "local disturbances" are sometimes of sufficient intensity to produce *local magnetic poles*, such as have been found by observation near Juneau, Alaska, and between Kursk and Odessa, in Russia.

It is usual to represent the distribution of the earth's magnetism graphically by means of *isogonic*, *isoclinic*, and *iso-dynamic* charts, on which are shown lines of equal declination, equal inclination, or equal intensity. For the construction of such charts many observations are required in order that the irregular distribution may be represented properly, and it is the usual experience that the addition of new observations brings out new irregularities. Inasmuch as the earth's magnetism is undergoing constant change, its distribution is different for different epochs, and a knowledge of the amount of change from one year to another is necessary before the results of observations made at different times can be reduced to the year for which it is desired to construct an iso-magnetic chart.

VARIATIONS OF THE EARTH'S MAGNETISM.

The continual change to which the earth's magnetism is subject has been analyzed in various ways and shown to be the resultant effect of several more or less systematic variations combined with irregular disturbances, which from time to time attain considerable magnitude, constituting what are known as *magnetic storms*. These "storms" occur at irregular intervals and may last only a few hours or several days and sometimes attain an intensity sufficient to produce a range of 1° or 2° in declination and of 2 or 3 per cent in the horizontal intensity. They usually occur almost simultaneously over the entire surface of the globe, and often accompany auroral displays and the appearance of large spots on the sun. The occurrence of a storm during observations can usually be detected by the erratic behavior of the magnet or needle, and calls for a repetition of the observations after the storm has subsided.

Of the systematic variations the largest and most important is the *secular variation*, so called because it requires centuries for its full development. While magnetic observations do not as yet cover a sufficiently long term of years to warrant a definite conclusion, yet the evidence is strong that at least for the *direction* of the earth's field the secular variation is of a periodic character, i. e., that the change with lapse of time does not go on indefinitely in one direction. Eventually a turning point is reached. In the case of the declination, numerous series of observations are available which are of sufficient extent to include one and in some cases probably two such turning points. Tables showing the secular change of the magnetic elements in the United States will be found on pages 114–119 of "United States Magnetic Tables and Magnetic Charts for 1905."

Of the periodic variations having for periods a year, a solar day and a lunar day, the only one of sufficient magnitude to be of practical importance is the *solar-diurnal variation*, or as it is usually designated, *diurnal variation*. Tables IX, X, and XI show the diurnal variation of declination, dip, and horizontal intensity at four of the magnetic observatories of the Coast and Geodetic Survey, based upon several years' observations. They were condensed from tables giving the average diurnal variation for 10 selected days for each month of the years specified.

The diurnal variation appears to be closely associated with the position of the sun above the horizon. During the night hours there is normally little change in any of the three elements. The daily range is greater in years of maximum sun spot frequency than in years of minimum, and varies somewhat with the season.

From an inspection of Table IX it will be seen that for all of the observatories the diurnal variation of declination shows the same general characteristics; a well-marked maximum (easterly extreme) between 8 and 10 in the morning, and a well-marked minimum (westerly extreme) between 1 and 3 in the afternoon.

For dip and horizontal intensity, on the other hand, there is only one well-marked extreme, except in the case of the summer months at Cheltenham, and even there one extreme is much more pronounced than the other. In all cases this extreme occurs not far from noon, but at Sitka and Cheltenham it is a maximum for dip and a minimum for horizontal intensity, while for Honolulu and Porto Rico it is a minimum for dip and a maximum for horizontal intensity.

DERIVATION OF FORMULAS.

DETERMINATION OF THE TRUE MERIDIAN BY OBSERVATIONS OF THE SUN.

As the magnetic declination is the angle between the true meridian and the magnetic meridian, its measurement requires the determination of the direction of both of these planes. The direction of the magnetic meridian is obtained by means of a magnet free to rotate in a horizontal plane about a vertical axis. The direction of the true meridian may be determined by observations either of the sun or of a star, especially Polaris. In connection with magnetic work it is usually more convenient to make the observations in the daytime and the method in general use consists of a series of observations of the sun both morning and afternoon, each observation comprising a measure of the altitude of the sun and the angle between it and a reference (azimuth) mark, and a record of the time. The computation of the azimuth of the sun and the local mean time from observations of this character involves the solution of the spherical triangle defined by the pole, the zenith, and the sun, the three sides being known. The fundamental formulas of spherical trigonometry have been transformed to fit this special case as follows:

When the sides of a spherical triangle are known the angles may be computed by formulas of the form:

$$\tan^2 \tfrac{1}{2} A = \frac{\sin(s_1 - b)\sin(s_1 - c)}{\sin s_1 \sin(s_1 - a)}$$

in which $2s_1 = a + b + c$.

In Figure 1 let Z P S represent the triangle defined by the zenith, the pole, and the sun.

SP $= a = 90° - \delta = p$ given in the Ephemeris of the sun.

SZ $= b = 90° - h$ determined by observation.

PZ $= c = 90° - \phi$ determined by observation.

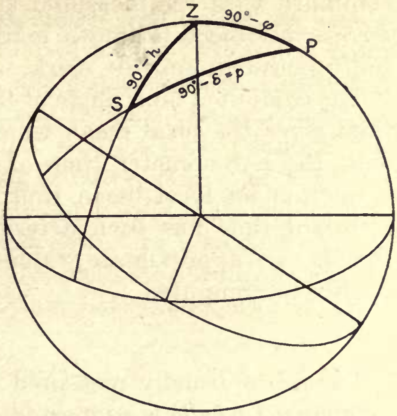

FIG. 1.—Fundamental spherical triangle.

The angle SZP $= A_n$ is the angle between the true meridian and the vertical plane through the sun and is therefore the azimuth of the sun counted from the North. The angle SPZ $= B$ is the hour angle of the sun, t. Substituting the values of a, b, and c in the formula and letting $2s = p + h + \phi$, the following transformations may be made:

$$2s_1 = 180° + p - h - \phi = 180° + 2p - 2\,s$$

$$s_1 = 90° + p - s = 90° - (s - p)$$

$$(s_1 - a) = 90° + p - s - p = 90° - s$$

$$(s_1 - b) = 90° + p - s - 90° + h = p + h - s = (s - \phi)$$

$$(s_1 - c) = 90° + p - s - 90° + \phi = p + \phi - s = (s - h)$$

$$\tan^2 \tfrac{1}{2} A_n = \frac{\sin\,(s - \phi)\,\sin\,(s - h)}{\cos s \cos\,(s - p)}$$

As it is usual to reckon azimuths from the south, substitute

$$180° - A_s = A_n$$

and the equation may be written in the form:

$$\text{ctn}^2 \tfrac{1}{2} A_s = \sec s \sec\,(s - p)\,\sin\,(s - \phi)\,\sin\,(s - h)$$

A similar transformation of the equation for the angle $B = t$ gives

$$\tan^2 \tfrac{1}{2}\,t = \cos s \sin\,(s - h)\,\csc\,(s - \phi)\,\sec\,(s - p)$$

and by combination with the equation for $\text{ctn}^2 \tfrac{1}{2} A_s$:

$$\tan^2 \tfrac{1}{2}\,t = \frac{\sin^2\,(s - h)\,\sec^2\,(s - p)}{\text{ctn}^2 \tfrac{1}{2} A_s}$$

and

$$\tan \tfrac{1}{2}\,t = \frac{\sin\,(s - h)\,\sec\,(s - p)}{\text{ctn}\,\tfrac{1}{2} A_s}$$

a very convenient form when the azimuth and hour angle are to be computed from the same set of observations.

The computed angle between the sun and the true south meridian combined with the measured angle between the sun and a selected terrestrial object (azimuth mark) gives the angle between the true south meridian and the mark, or the true azimuth of the mark.

The computed hour angle of the sun combined with the equation of time gives the local mean time of observation, and this compared with the chronometer time of observation gives the chronometer correction on local mean time. If the chronometer correction on standard time has been determined by means of telegraphic time signals, an approximate value of the longitude of the place can readily be computed.

DIP.

The dip is usually measured by means of a dip circle in which a magnetized needle is supported so as to be free to rotate in a vertical plane. A steel axle through the center of gravity of the needle terminates in finely ground pivots which rest on agate knife edges. The angle of dip is measured on a graduated circle concentric with the axle of the needle. In order to measure the angle of dip directly, the needle must swing in the magnetic meridian. The observed angle of inclination in any other plane will be too large, as will be seen from the following considerations. In the magnetic meridian the horizontal and vertical components of the total intensity are H and Z, and $Z = H \tan I$. In a plane making an angle α with the magnetic meridian, the components are $H \cos \alpha$ and Z, and $Z = H \cos \alpha \tan I_a$. Hence $\tan I = \cos \alpha \tan I_a$. As the cosine of an angle is always less than unity, I_a is always greater than I. This formula may be used to compute the true dip from observations out of the meridian, provided the angle α is known. The equation may be written in the form $\operatorname{ctn} I_a = \operatorname{ctn} I \cos \alpha$. Let $\alpha = 90°$; then $\operatorname{ctn} I_a = 0$ and $I_a = 90°$. That is, when the instrument is in the magnetic prime vertical the dip needle stands vertical, a fact which furnishes a simple method for setting the instrument in the magnetic meridian when a compass attachment is not available for the purpose. Extreme accuracy in the determination of the magnetic meridian is not required, as will be seen if α be computed from the above formula assuming $I = 45°$ and $I_a = 45° \, 00'.1$, the resulting value of α being $37'$. That is, unless the instrument is more than $30'$ out of the magnetic meridian, the effect on the dip is not as much as $0'.1$.

The true dip may be obtained by combining observations in two planes at right angles to each other.

For $$\operatorname{ctn} I_a = \operatorname{ctn} I \cos \alpha$$

and $$\operatorname{ctn} I_{(90°-\alpha)} = \operatorname{ctn} I \cos (90 - \alpha) = \operatorname{ctn} I \sin \alpha$$

Hence $$\operatorname{ctn}^2 I_a + \operatorname{ctn}^2 I_{(90°-\alpha)} = \operatorname{ctn}^2 I$$

The ideal dip-needle would be perfectly symmetrical in size and mass with respect to the axis of its pivots, but this condition can not be exactly attained by the maker, and subsequent use of the needle is liable to increase the divergence from this ideal condition. Most of the errors due to lack of symmetry and adjustment are eliminated by reversal of instrument and needle and reversal of the polarity of the needle. Yet it will usually be found that different values of dip are obtained before and after reversing polarity, indicating that the needle would not exactly balance if demagnetized. This lack of balance may be ascribed without material error * to a small weight p in the longitudinal axis of the needle at a distance d from the axis of the pivots. The equations of equilibrium before and after reversal of polarities will be:

$$pd \cos I_n = FM \sin (I - I_n)$$

and

$$pd \cos I_s = FM \sin (I_s - I)$$

assuming that the magnetic moment M of the needle is the same before and after reversal.

Hence
$$\frac{\cos I_n}{\cos I_s} = \frac{\sin(I - I_n)}{\sin(I_s - I)} = \frac{\sin I \cos I_n - \cos I \sin I_n}{\sin I_s \cos I - \cos I_s \sin I}$$

Clearing of fractions and dividing by $\cos I_s \cos I_n \cos I$,

$$\tan I_s - \tan I = \tan I - \tan I_n$$

$$\tan I = \frac{\tan I_n + \tan I_s}{2}$$

That is, where the observations give different values of dip before and after reversal of polarities, the mean of the two quantities does not give the true dip. Instead, the angle must be found whose tangent is the mean of the tangents of the observed angles. To avoid the necessity of making this computation for each observation, Table VIII has been prepared, giving the correction required by the dip obtained by using the formula

$$I = \frac{I_n + I_s}{2}$$

For example, if the observed dip was 72° 15′.0 before reversal of polarities and 72° 45′.0 afterwards, the true dip would be 72° 30′.0 + 0′.2 = 72° 30′.2.

At a magnetic observatory an *earth inductor* is usually provided for determining the dip. With this instrument more accurate results may be obtained in less time than with a dip circle. The operation of the earth inductor is based on the principle that when a closed circuit is revolved in a magnetic field, electric currents are induced unless the axis of rotation is tangent to the lines of force of the field.

* The needle is so long compared with its width that the lack of symmetry with respect to the longitudinal axis is not apt to be appreciable.

After the instrument has been leveled and placed with the axis of rotation in the magnetic meridian, the coil is rotated and its inclination is changed until the induced current becomes zero as shown by a galvanometer placed in the circuit. The angle of inclination of the coil, or dip, is then read off on the vertical circle.

Numerous comparisons of dip circles with each other and with earth inductors have established the fact that, in spite of every refinement of adjustment and care in observing, different dip circles give different results and nearly all require corrections to reduce to the more accurate earth inductor results. This is probably due in many cases to irregularity of pivots of the needles and sometimes to slight impurities in the metal entering into the make-up of the instrument. While the effect of either of these causes would be different for different angles of dip, it is the practice in the Coast and Geodetic Survey to assume a uniform correction for the limited range of dip involved in a season's work.

In the case of two dip circles which were used over a wide range of dip and showed large and variable corrections, analytical expressions of the form

$$\Delta I = \frac{x}{F} + y\,\frac{\sin I}{F} + z\,\frac{\cos I}{F}$$

were derived, from which to compute the required corrections, based on the assumption that the varying corrections were to be ascribed to the effect of the metal composing the instrument.

HORIZONTAL INTENSITY.

Up to the time of Gauss all measures of horizontal intensity were relative, and consisted in comparing the times of oscillation at different places of a magnet rotating in the horizontal plane about a vertical axis. Assuming the magnetic moment of the magnet to be constant, the horizontal intensity is inversely proportional to the square of the time of oscillation. As a matter of fact, all magnets tend to lose their magnetism gradually, but this decrease of magnetic moment was determined and allowed for approximately by observing at a base station both at the beginning and end of a voyage or a season's work.

Gauss conceived the idea of combining with the oscillations a set of observations in which the intensity magnet is used to deflect an auxiliary magnet, and thus determine the horizontal intensity absolutely, and this is the method in general use at the present day. Two distinct operations are involved: *Oscillations*, which serve to determine the product of the magnetic moment of the magnet and the horizontal intensity; *deflections*, from which the ratio of the same two quantities is obtained.

OSCILLATIONS.

If a magnet be free to rotate about a vertical axis through its center of gravity, it will come to rest with its magnetic axis in the magnetic meridian. If it be turned out of that plane and then released it will oscillate as a horizontal pendulum under the influence of the earth's magnetism, the amplitude of its swing gradually diminishing until it finally comes to rest again in the magnetic meridian. If T be the time of one oscillation; i. e., the time between two successive transits (in opposite directions) across the meridian, K the combined moment of inertia of the magnet and stirrup about the axis of rotation, and M the magnetic moment of the magnet, the equation of motion for a pendulum becomes:

$$H\,M=\frac{\pi^2 K}{T^2}$$

subject, however, to certain corrections as explained below.

Reduction to infinitesimal arc.—The above formula is based on the assumption that the arc of vibration is infinitesimal. For a finite arc the observed time of one oscillation must be diminished by a small amount, the corrective factor being $\left(1-\frac{a'a''}{64}\right)$, in which a' and a'' are the initial and terminal arcs of vibration, expressed in radians, or simply $\left(1-\frac{a^2}{64}\right)$, a being the average arc of vibration. From the adjoining table it will be seen that for an average arc of 3° this correction amounts to only 1 part in 25000, and as the arc of vibration need never exceed this amount, and in the majority of magnetometers is still more restricted by the limits of the scale of the magnet, this correction is in general negligible.

a	$\frac{a^2}{64}$
1°	0.00000
2	02
3	04
4	08
5	12

Correction for rate of chronometer.—The observed time of one oscillation must be corrected for the rate of the chronometer used. If r be the *daily* rate of the chronometer in seconds, plus when losing and minus when gaining, the observed value of T must be multiplied by the factor $\left(1+\frac{r}{86400}\right)$ or T must be increased by $0.0000116\,Tr$. Values of this expression for different values of T and r are given in Table V.

Correction for torsion.—The earth's magnetism is not the only force acting to cause the oscillations. It is usual to suspend the magnet by one or more silk fibers or by a very fine wire or metallic ribbon, the torsion of which must be taken into account. The ratio between the force of torsion and the horizontal intensity may be determined in the following manner: When the magnet is at rest in the magnetic meridian, if the upper end of the suspension fiber be turned through any angle, say 90°, the magnet will be turned out of the meridian through a small angle h (expressed in minutes) on account of the torsion of the fiber. The equation of equilibrium between the two forces for a twist of 90° is:

$$C\,(5400-h)=M\,H\sin h \qquad \text{or } C=\frac{M\,H\sin h}{(5400-h)}$$

in which C is the force of torsion per minute of arc. Experiments have shown that the force of torsion is approximately proportional to the amount of twist. In the case in point the upper end of the fiber is turned through 5400′, but the lower end is turned in the same direction through the angle h, so that the amount of twist is $(5400-h)$.

When during oscillations the magnet makes any angle, as θ, with the meridian the force exerted by the earth's magnetism to pull it back into the meridian is $M\,H\sin\theta$, and the force of torsion acting in the same direction is $\frac{\theta M\,H\sin h}{5400-h}$ and the resultant of the two:

$$M\,H\sin\theta+\frac{\theta M\,H\sin h}{5400-h}=M H\sin\theta\left[1+\frac{\theta\sin h}{\sin\theta\,(5400-h)}\right]$$

$$=M H\sin\theta\left[1+\frac{h}{5400-h}\right],$$

since both h and θ are small. Hence, in the oscillation formula,

$$M H\left(1+\frac{h}{5400-h}\right)=M\,H\left(\frac{5400}{5400-h}\right)$$

must be substituted for $M H$ in order to take into account the effect of torsion. Values of the logarithm of $[5400\div(5400-h)]$ for different values of h are given in Table VI, but for small values such as are usually experienced in magnetometers the logarithm of this factor may be assumed proportional to h, i. e.:

$$\log 5400-\log\,(5400-h)=h\,[\log\,5400-\log\,(5400-1)]=h\,[0.00008].$$

Induction.—When a magnet is placed in a magnetic field its magnetism is temporarily increased by induction by an amount proportional to the strength of that component of the field which is parallel to the axis of the magnet. In the case of the oscillating magnet, its magnetic moment is increased from M to $(M+\mu\,H)$ or $M\left(1+\mu\frac{H}{M}\right)$, μ being the induction factor.

Temperature correction.—As the magnetic moment of a magnet changes with change of temperature, increasing as the temperature decreases and vice versa, and as in general the temperature of the magnet is different for the two sets of observations, deflections and oscillations, it is necessary to allow for this difference in temperature before combining the two equations to compute H and M. If M

and M' be the magnetic moments at temperatures t and t' respectively, then the temperature coefficient, q, is represented by the formula

$$q=\frac{M'-M}{t-t'}\,\frac{2}{(M+M')}$$

As $(M'-M)$ is usually very small as compared with M and M' it will not introduce a material error to substitute either M or M' for $\frac{M+M'}{2}$. The change in M with change in temperature may then be computed by the formula

$$M'=M[1+(t-t')q]$$

If t' be the temperature of the magnet during oscillations and t the temperature during deflections, then the formula gives the magnetic moment at the temperature of the oscillations expressed in terms of the moment at the temperature of deflections.

The corrections for rate of chronometer and reduction to infinitesimal arc may be readily applied to the observed value of T. It is more convenient to correct for torsion, induction, and change of temperature by the addition of three factors to the oscillation formula so that it becomes:

$$HM\left(\frac{5400}{5400-h}\right)\left(1+\mu\frac{H}{M}\right)\left(1+(t-t')q\right)=\frac{\pi^2K}{T^2}$$

or

$$HM=\pi^2K\div T^2\left(\frac{5400}{5400-h}\right)\left(1+\mu\frac{H}{m}\right)\left(1+(t-t')q\right)$$

DEFLECTIONS.

A magnet free to turn about a vertical axis will come to rest with its magnetic axis in the magnetic meridian if acted on by the earth's magnetism alone. If a second magnet be brought near to the suspended magnet, the latter will be deflected out of the magnetic meridian by an amount depending upon the relative strength of the two forces acting upon it. The law of the action between two magnets under these conditions was developed by Gauss for the special cases where the two magnets lie in the same horizontal plane, (1) with the axis of the deflecting magnet in the magnetic prime vertical through the center of the suspended magnet and (2) with the center of the deflecting magnet in the magnetic meridian through the center of the suspended magnet and with its axis in the magnetic prime vertical. Lamont later extended the discussion to the cases where the axes of the deflecting and suspended magnets are at right angles to each other, (1) the deflector being to the east or west and (2) the deflector being to the north or south. In 1890 Börgen developed the formula for the most general case, placing no restrictions upon the relative positions of the two magnets, and derived therefrom the forms applicable to the special cases already treated by Gauss and Lamont. Nearly all magnetometers of recent make are arranged with deflection bars attached at right angles to the telescope by which pointings are made on the suspended magnet, and hence the deflections are made in Lamont's first position. It will be sufficient for our purposes, therefore, to outline the method of deriving the deflection formula for that special case. Suppose that the suspended magnet $N_1 S_1$ is deflected out of the magnetic meridian through the angle u by the magnet N S, placed so that the prolongation of its magnetic axis passes through the center of $N_1 S_1$. Let m and m_1 and $2\,l$ and $2\,l_1$ be the pole strength and distance between poles of the two magnets and r the distance between their centers. Then the magnetic moments are $M=2ml$ and $M_1=2m_1l_1$. For an approximate solution of the problem, assume that l_1 is so small compared with r that the distances from the pole N to N_1 and S_1 may be taken as $(r+l)$ and from S to N_1 and S_1 as $(r-l)$. Then the force of attraction between S and N_1 is $\frac{m\,m_1}{(r-l)^2}$ and the turning moment is $\frac{m\,m_1\,l_1}{(r-l)^2}$. The force of repulsion between N and N_1 is $\frac{m\,m_1}{(r+l)^2}$ and the corresponding turning moment is $\frac{m\,m_1\,l_1}{(r+l)^2}$. The total turning moment resulting from the action between the two magnets is therefore:

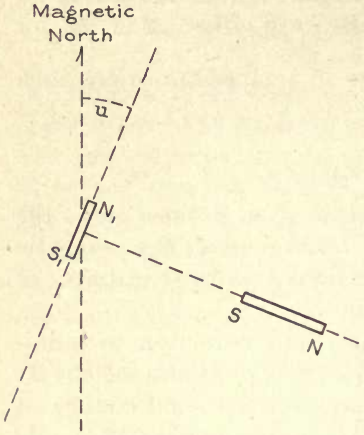

FIG. 2.—Position of magnets during deflections.

$$\frac{2\,m\,m_1\,l_1}{(r-l)^2}-\frac{2\,m\,m_1\,l_1}{(r+l)^2}=\frac{8\,m\,m_1\,l\,l_1\,r}{(r^2-l^2)^2}=\frac{2\,MM_1\,r}{(r^2-l^2)^2}$$
$$=\frac{2\,MM_1}{r^3}\left(1+\frac{2\,l^2}{r^2}+\frac{3\,l^4}{r^4}+\frac{4\,l^6}{r^6}------\right)$$

The rigorous solution, taking into account the pole distance of N_1S_1 yields an expression of the same form, namely: $\frac{2\,MM_1}{r^3}\left(1+\frac{P}{r^2}+\frac{Q}{r^4}+--\right)$ in which P, Q, and succeeding coefficients are functions of the dimensions of the two magnets and the distribution of their magnetism. The series converges so rapidly that the coefficients beyond Q need not be considered for properly chosen deflection distances.

The turning moment of the force tending to pull the suspended magnet back into the meridian is $H\,M_1 \sin u$, u being the angle of deflection. When the magnet is at rest the two opposing forces are equal and opposite,

hence
$$H M_1 \sin u = \frac{2MM_1}{r^3}\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$$

and
$$\frac{H}{M}=\frac{2}{r^3 \sin u}\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$$

P and Q are called the first and second distribution coefficients, and their values could be computed from the dimensions of the magnets by means of Börgen's formula, provided the ratio of distance between poles to length of magnet was known. This ratio is difficult to determine with accuracy and appears to be different for different magnets, so that only approximate results can be obtained by this method. Börgen concludes that on the average the pole distance is a little more than 0.8 the length of the magnet and, assuming that it is the same for both deflecting and suspended magnets, deduces the formulas:

$$P=2l^2-3l_1^2$$
$$Q=3l^4-15l^2l_1^2+\frac{45}{8}l_1^4$$

It is better, in view of the uncertainty of the above ratio, to depend on observations for the determination of P and Q. It is evident that if deflection observations are made at three distances there will result three equations from which the three unknowns $\frac{H}{M}$, P, and Q can be computed, as explained later.

The above equations for P and Q are useful, however, to determine approximately the relative length of the two magnets which will make P or Q zero.

When $P=0$ $\quad 2l^2=3l_1^2$ $\quad$ and $l=1.225l_1$.

When $Q=0$ $\quad 3l^4-15l^2l_1^2+\frac{45}{8}l_1^4=0$ and $l=2.15l_1$.

In deriving the deflection formula given above no account has been taken of the effect of induction upon the magnetic moment M of the deflecting magnet. It will readily be seen that the south end of the deflecting magnet will always be inclined to the north of the magnetic prime vertical whether it is placed to the east or west of the suspended magnet or with its north end east or west, and the effect of induction will therefore always correspond to a decrease in its magnetic moment. As already stated, the induction is proportional to the strength of that component of the earth's field which is parallel to the axis of the magnet, in this case $H \sin u$. Hence the moment of the deflecting magnet when the suspended magnet is deflected through the angle u is really $(M-\mu\ H \sin u)$ instead of M, μ being the induction factor of the magnet. As $\mu\ H \sin u$ is always very small in comparison with M, we may substitute for $H \sin u$ its approximate value $\frac{2M}{r^3}$

and $$(M - \mu H \sin u) = \left(M - \frac{2M\mu}{r^3}\right) = M\left(1 - \frac{2\mu}{r^3}\right)$$

Making this correction to the deflection formula, it becomes:

$$\frac{H}{M\left(1 - \frac{2\mu}{r^3}\right)} = \frac{2}{r^3 \sin u}\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)$$

or $$\frac{H}{M} = \frac{2}{r^3 \sin u}\left(1 - \frac{2\mu}{r^3}\right)\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)$$

It is probable that the distribution of the magnetism of a magnet changes somewhat in the course of time, and consequently P, Q, and μ are subject to change, but results show that for a season's work, or even longer, they may be considered constant without materially increasing the uncertainty of the results, especially when the magnets have become so well seasoned that the loss of magnetism is very slow. The deflection formula may then be written:

$$\frac{H}{M} = \frac{C}{\sin u}$$

in which $C = \frac{2}{r^3}\left(1 - \frac{2\mu}{r^3}\right)\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)$ and is constant for a particular deflection distance and a particular temperature. Its variation with temperature may be readily computed from the coefficient of expansion of the material, usually brass, of which the deflection bars are made, since r is the only quantity in the second member which varies with temperature.

The oscillations give the product of H and M

$$HM = \pi^2 K \div \left[T^2\left(\frac{5400}{5400-h}\right)\left(1 + \mu\frac{H}{M}\right)\left(1 + (t - t')q\right)\right]$$

and the deflections give their ratio:

$$\frac{H}{M} = \frac{2}{r^3 \sin u}\left(1 - \frac{2\mu}{r^3}\right)\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)$$

from which the values of H and M can be readily computed, if we assume that the values are the same for the two sets of observations. So far as M is concerned this is a safe assumption, provided allowance is made for the change in temperature between the two classes of observations, as has been done in the formula for HM. Experience shows that a magnet loses its magnetism quite rapidly for a short time after magnetization, but soon settles down to a condition of very slow change, inappreciable for the period covered by a set of intensity observations. Exception should be made of the sudden loss of magnetism resulting from a shock such as would be caused by dropping the magnet or from bringing it into contact with another magnet.

In the case of H there is constant change, usually small in extent during the time covered by a set of observations, but at times exceeding in amount the error of observation. To minimize the effect of this variation, the observations are usually arranged in the order: Oscillations, deflections, deflections, oscillations. The following considerations show that small changes in H such as are exceeded only at times of severe magnetic storms have no appreciable effect on the result. For suppose H_o and H_d are the values of H at the time of oscillations and deflections, respectively, and let $H_d = H_o + \Delta H$. The combination of the observations on the assumption that $H_o = H_d$ would give the value $H = \sqrt{H_o\ H_d} = \sqrt{H^2_o + H_o\ \Delta H}$. The quantity under the radical differs from $\left(H_o + \frac{1}{2}\Delta H\right)^2$ by $\frac{\Delta H^2}{4}$, a quantity so small as to be negligible except in the case of a severe magnetic storm. But $H_o + \frac{1}{2}\Delta H = \frac{H_d + H_o}{2}$. Hence it is evident that the assumption of no change in H between the deflection and oscillation observations gives a value of H which is the mean for the period covered by the observations. To show the effect in an extreme case, suppose $\Delta H = 0.05H$, a range seldom reached in the course of a magnetic storm, then

$$\frac{H_d + H_o}{2} - \sqrt{H_d\ H_o} = 0.0012\ H.$$

Under such conditions the magnet would be so disturbed as to render accurate observations impossible.

TOTAL INTENSITY.

Under certain conditions it is inconvenient or impossible to use the above method for determining the horizontal intensity. As the magnetic pole is approached the horizontal intensity becomes so small that the method fails for lack of accuracy. On shipboard the motion of the vessel precludes the use of a fiber suspension, which is essential to accurate oscillation observations. At times it is necessary to reduce the instrumental equipment of a party as much as possible. In such cases use may be made of the method devised by Dr. E. Lloyd to determine the total intensity by means of a dip circle. While inferior in accuracy under ordinary conditions to the method of determining the horizontal intensity with a magnetometer, yet with a good dip circle carefully handled it will usually yield very satisfactory results. (See App. 3, C. & G. S. Report for 1905, p. 114.)

The method involves two operations, during both of which the dip circle is so placed that the suspended needle swings in the magnetic meridian: *First,* the measure of the angle of inclination with a needle having a weight in the south end (in north magnetic latitudes); *second,* the measure of the angle through which a second needle is deflected by the loaded needle, when the latter is placed at right angles to it in the place provided for the purpose between the reading microscopes, with the axes of rotation of the two needles lying in the same straight line. In the first case the earth's magnetism acting on the loaded (intensity) needle is opposed to the force of gravity acting on the weight. In the second case the force exerted by the intensity needle on the suspended needle is opposed to the earth's magnetism.

Let I' = the dip with loaded needle, considered positive when the south end is above the horizon. Then the angle through which the needle is turned by the weight is $u' = I - I'$

u = Deflection angle.

M = Magnetic moment of the intensity needle.

M_1 = Magnetic moment of the second needle.

k = Mass of the weight.

R = Distance of weight from the axis of rotation.

The equation of equilibrium for the dip with loaded needle is:

$$kR \cos I' = FM \sin u'$$

For the deflection observations, the equation is:

$$k_1 MM_1 = FM_1 \sin u$$

in which k_1 is a factor depending upon the distance between the needles and the distribution of their magnetism. Combining the two equations:

$$kk_1 RMM_1 \cos I' = F^2 MM_1 \sin u \sin u'$$

Let $$kk_1 R = C^2$$

Then $$C^2 \cos I' = F^2 \sin u \sin u'$$

$$F = C\sqrt{\cos I' \csc u \csc u'}$$

$$C = F\sqrt{\sec I' \sin u \sin u'}.$$

When the above observations are made at a place where the dip and horizontal intensity (and hence also the total intensity) are known, the value of C can be computed. Knowing C the value of F at any other place can be determined by observation. As the factor C involves the mass of the weight and its distance from the axis of rotation and also the distribution of magnetism in the needles, it is necessary to guard against change, in the interval between the standardization observations and those at other places. The weight should be left in position and care should be taken not to change the magnetic condition of the needles. Hence they *must not be remagnetized* in the course of a season's work.

DETERMINATION OF THE CONSTANTS OF A MAGNETOMETER.

The two formulas used in the determination of H and M from observations of oscillations and deflections involve a number of factors which must be determined by special observations or otherwise before they can be used, namely, *moment of inertia*, *temperature coefficient*, *induction coefficient*, and *distribution coefficients*, as well as the *deflection distances.*

MOMENT OF INERTIA.

The magnets of most magnetometers are of the *collimator* type, a hollow steel cylinder closed at one end by a glass on which a scale is etched and at the other by a lens. There is thus introduced a lack of homogeneity which makes it impracticable to compute K, the moment of inertia of the magnet, from the dimensions of its component parts. Moreover the magnet is usually supported by means of a stirrup of more or less complex form, and it is the moment of the magnet and stirrup combined which is involved in the formula. It is usual, therefore, to determine the moment of inertia by means of an auxiliary weight of nonmagnetic material and of regular form of which the moment of inertia can be readily computed from its dimensions and mass. A truly turned bronze ring or a circular cylinder of about the same mass as the magnet are the forms commonly employed. For a ring the moment of inertia is given by the formula:

$$K_1 = \frac{W}{8}(d^2 + d_1^2)$$

in which d and d_1 are the inner and outer diameters and W is the mass. For a cylinder the formula is:

$$K_1 = \frac{W}{48}(4l^2 + 3d^2)$$

in which l is the length and d the diameter. To find the value of K_1 for any other temperature than the one at which the dimensions were measured, the average coefficient of expansion of bronze, 0.000018 for 1° C., may be used, unless a special determination has been made for the weight in question. It will be seen that 2 log (1.000018) = 0.000016 is the corresponding change in log K_1 for 1° change in the temperature of the inertia weight.

If in addition to oscillations with the magnet alone, observations are made with the weight added, two equations will result:

$$HM = \pi^2 K \div \left[T^2 \left(\frac{5400}{5400 - h} \right) \left(1 + \mu \frac{H}{M} \right) \left(1 + (t - t')q \right) \right]$$

$$HM = \pi^2 (K + K_1) \div \left[T_1^2 \left(\frac{5400}{5400 - h_1} \right) \left(1 + \mu \frac{H}{M} \right) \right]$$

Hence

$$\frac{K}{T^2 \left(\frac{5400}{5400 - h} \right) \left(1 + (t - t')q \right)} = \frac{K + K_1}{T_1^2 \left(\frac{5400}{5400 - h_1} \right)}$$

supposing H to remain constant and allowing for change of M with change of temperature, t' being the temperature of the magnet during oscillations without the weight and t the temperature during oscillations with the weight.

$$\text{Let} \quad (T)^2 = T^2\left(\frac{5400}{5400-h}\right)\left(1+(t-t')q\right) \text{ and } (T_1)^2 = T_1^2\left(\frac{5400}{5400-h_1}\right)$$

$$\text{Then} \qquad \frac{K}{(T)^2} = \frac{K+K_1}{(T_1)^2} \quad \text{and} \quad K = \frac{(T)^2 K_1}{(T_1)^2-(T)^2}.$$

That is, this simple formula may be used if the observed values of T^2 and T_1^2 are corrected for torsion and the former is reduced to the temperature of the latter. The following arrangement of the observations will practically eliminate small changes in H:

Begin with a set of oscillations without the inertia weight, determining the torsion factor with a torsion weight of the same mass as the magnet. Then make a set of oscillations with the inertia weight added, determining the torsion factor again, but with a torsion weight of the same mass as the magnet and inertia weight combined. Continue making sets of oscillations alternately with and without the weight, ending the series with a set without the weight. Determine the torsion factor again with the last set of each class of oscillations. Each set of oscillations will consist of 8 independent determinations of the time of a selected number of oscillations. The first set of oscillations without the weight and the first half of the second set, combined with the intervening set with the weight, give one value of K. The second half of the second set without the weight and the first half of the third set combined with the second set with the weight give another value of K, and so on. Five of these independent determinations will usually give a satisfactory mean value of K. The change in K with temperature is a function of the temperature coefficient of steel, which may be taken as 0.000011 for 1° C. For a change of 1° in temperature the corresponding change in log K is

$$2 \log (1.000011) = 0.00001.$$

The method of separating the intermediate sets of oscillations without the weight into two parts is shown in the following example. A convenient form of computation is also given.

MOMENT OF INERTIA. OSCILLATIONS, without weight.
Station, Cheltenham, Md. Date, April 28, 1909.
Magnetometer No. 26. Magnet 26 L.
Chronometer No. 1107, daily rate gaining $8^s.1$ on mean time.

Number of oscillations.	Chronometer time.	Temp. t'	Extreme scale readings.		Time of 70 oscillations.
	h. m. s.				
0	15 25 15.7	18.2	−22.8	+22.8	
7	25 52.6				
14	26 29.5				
21	27 06.4				
28	27 43.2				
35	28 20.1				
42	28 57.1				
49	29 34.0	18.1			*m. s.*
70	15 31 24.8				6 09.1
77	32 01.6				09.0
84	32 38.5				09.0
91	33 15.3				08.9
98	33 52.2				09.0
105	34 29.0				08.9
112	35 06.0				08.9
119	35 42.9	18.1	−17.9	+17.9	08.9
	Means	18.13			{ 6 09.00 6 08.925

		First half.	Second half.
t (preceding set) = 17.°97	Time of 1 oscil.	*s.* 5.27143	*s.* 5.27036
$t-t'$ = −0.°16	Corr'n for rate	−49	−49
t (following set) = 18.°27	T	5.27094	5.26987
$t-t'$ = +0.°14	Log T^2	1.44378	1.44360
	" $\left(\frac{5400}{5400-h}\right)$	60	60
	" $[1+(t-t')q]$	− 5	+ 4
	" $(T)^2$	1.44433	1.44424
	$(T)^2$	27.818	27.812

COMPUTATION OF MOMENT OF INERTIA.

Cheltenham, Md. April 28, 1909.
Magnetometer No. 26. Inertia ring A.

Chron. time.	Temp.	T_2	T_1^2 and $T_1^2-T^2$	Log T^2 and log K_1	Log T^2K_1 and $\log(T_1^2-T^2)$	log K	log K_{20}
h. m.	°						
14 32		27.824		1.44437			
15 05	17.97	[27.821]	43.409	2.47036	3.91473		
15 29		27.818	15.588		1.19279	2.72194	2.72196
15 32		27.812		1.44429			
15 52	18.27	[27.816]	43.419	2.47036	3.91465		
16 11		27.820	15.603		1.19321	2.72144	2.72146
16 14		27.841		1.44433			
16 36	18.53	[27.818]	43.416	2.47037	3.91470		
16 52		27.794	15.598		1.19307	2.72163	2.72164
16 55		27.820		1.44439			
17 12	18.95	[27.822]	43.409	2.47037	3.91476		
17 27		27.824	15.587		1.19276	2.72200	2.72201
17 30		27.835		1.44458			
17 48	19.33	[27.834]	43.433	2.47038	3.91496		
18 19		27.834	15.599		1.19310	2.72186	2.72187
18 22		27.846		1.44476			
18 40	20.15	[27.846]	43.445	2.47039	3.91515		
19 06		27.845	15.599		1.19310	2.72205	2.72205
						Mean	2.72183

K=527.02 at 20° C.

When the weight is a cylindrical bar, there is usually a place provided in the stirrup for suspending it above or below the magnet. When a ring is used, it must be balanced on top of the magnet, so as to be horizontal and with its center in the line of suspension. To facilitate placing it in this position, a wooden block is provided having a socket in which the magnet will fit with its upper surface even with the surface of the block. Suitable marks on the block indicate the position in which the ring must be placed in order to be symmetrical with respect to the center of the magnet. It will, in general, be necessary to increase the number of suspension fibers in order to support the increased weight.

The moment of inertia of a magnet will be affected by any change in its dimensions or mass. A screwing up or unscrewing of one of the end cells would produce a slight change of length. The removal of a large amount of accumulated rust would produce an appreciable change of mass. The magnet must be carefully protected therefore from these or similar changes, and in case such a change should take place its moment of inertia must be redetermined.

TEMPERATURE COEFFICIENT.

When the temperature of a magnet increases, its magnetic moment decreases, and vice versa. Experiments have shown that the rate of change is not uniform, but increases with increase of temperature. In view of the small change of temperature usually experienced during a set of horizontal intensity observations and the partial elimination of its effect by a symmetrical arrangement of oscillations and deflections, no material error will be introduced by the assumption that the rate of change is uniform for ordinary temperatures.

If M and M' be the values of the magnetic moment of a magnet at temperatures t and t' and q be the temperature coefficient:

$$q = \frac{M - M'}{M(t' - t)} \qquad \text{and} \qquad M' = M(1 + (t - t')q)$$

From an inspection of the oscillation and deflection formulas, it will be seen that if two sets of observations of either class be made at different temperatures, the value of q may be computed, provided means are taken to allow for change of H. At an observatory this may readily be done with the aid of the continuous record of the magnetograph. In any case, the effect may be nearly eliminated by observing alternately at high and low temperatures and combining two sets of observations at about the same temperature with an intervening set at a different temperature. Care must be taken to maintain a given temperature for a sufficient time to make sure that the magnet and thermometer are at the same temperature, and rapid changes should be avoided. If both oscillations and deflections are made at high and low temperatures, the change in H is obtained from the observations themselves. If the observations are made in a room which can be heated and cooled artificially, no special apparatus is required. Otherwise the value of q is most conveniently determined by deflection observations, the deflecting magnet being surrounded by a water jacket, which may be filled alternately with hot and cold water. In this case allowance must be made for the effect of change of temperature upon the length of the bar.

The computation of q may be conveniently made by logarithms, bearing in mind that for our purposes $\log\,(1 + (t - t')q)$ may be replaced by $(t - t')\,[\log\,(1 + q)]$ without materially affecting the results.

$$\log M' = \log M + (t - t')\log\,(1 + q)$$

$$\log\,(1 + q) = \frac{\log M' - \log M}{t - t'}$$

If special deflection observations have been made, they give directly

$$\log \frac{H}{M} - \log \frac{H}{M'} = \log M' - \log M$$

It will be sufficient to use the approximate values of $\frac{H}{M}$ and $\frac{H}{M'}$, namely, $\frac{2}{r^3 \sin u}$ and $\frac{2}{r_1^{\,3} \sin u_1}$ when the induction and distribution coefficients are not known, r and r_1 being the values of the deflection distance at temperatures t and t'.

A check on the correctness of an adopted value of q may be obtained from the values of log M determined in the course of a season's work. When all the values have been reduced to a common temperature, they should show a fairly uniform decrease with lapse of time. An error in the adopted temperature coefficient would be indicated by deviations from a uniform change which conform in general with the changes in temperature.

INDUCTION COEFFICIENT.

When a magnet is placed in a magnetic field its magnetic moment is temporarily changed by induction by an amount which is proportional to the component of the field directed parallel to the axis of the magnet. The rate of change, i. e., the ratio of the moment of the magnet to the change produced by a unit field, is called the *induction coefficient*, h. The change in the magnetic moment M of a magnet placed parallel to a field of intensity H would be hMH, or μH, $\mu = Mh$, called the induction factor, being the change in the magnetic moment produced by a field of unit intensity. The induction coefficient is not constant, but varies with the strength of magnetization of the magnet. It is different also according as the induction tends to increase or decrease the magnetic moment; the more strongly a magnet is magnetized, the less susceptible it becomes to increase of magnetization by induction, but the more susceptible to decrease. In the oscillation observations induction increases the magnetic moment of the magnet, and the *induction factor* may be taken as constant. In the deflection observations the effect of induction is to reduce the magnetic moment of the magnet, but the magnet is in general so nearly in the prime vertical that the effect is very small, and hence the assumption that the induction factor is the same as for the oscillations does not introduce an appreciable error.

Of the various methods for determining the induction coefficient, the one devised by Lamont has been used exclusively by the Coast and Geodetic Survey. The magnet of which the induction coefficient is desired is used as a deflector with its axis vertical, in the vertical plane at right angles to the suspended magnet, but with its center some distance above or below the horizontal plane through that magnet. Observations are made first with north end up, magnet up, and then with north end down, magnet down. In the former position the magnetic moment of the magnet is decreased by induction, and in the latter is increased. If care is taken to maintain constant conditions, except for the inversion of the deflecting magnet, the change in the deflection angle will be a measure of the change in the magnetic moment due to the inductive effect of the vertical intensity, Z. In the first case

$$\frac{H}{M(1-hZ)}=\frac{C}{\sin u_1}$$

and in the second case

$$\frac{H}{M(1+hZ)}=\frac{C}{\sin u_2}$$

$$\frac{1+hZ}{1-hZ}=\frac{\sin u_2}{\sin u_1}$$

$$hZ=hH\tan I=\frac{\sin u_2-\sin u_1}{\sin u_2+\sin u_1}=\frac{\tan\frac{1}{2}(u_2-u_1)}{\tan\frac{1}{2}(u_2+u_1)}$$

$$h=\frac{1}{H\tan I}\cdot\frac{\tan\frac{1}{2}(u_2-u_1)}{\tan\frac{1}{2}(u_2+u_1)}$$

This method involves the assumption that the induction coefficient is the same whether it tends to increase or decrease the moment of the magnet. As the corrections for induction are very small, this is a safe assumption for all except the most refined observations.

As the induction coefficient is a very small quantity, the change in the deflection angle (u_2-u_1) is small also and a small error in observation or a small change in the relative position of the two magnets will materially affect the result. It is usual to extend the observations by varying the position of the deflecting magnet, as indicated in the following sample set, and also by making several sets using different horizontal and vertical distances. For making the observations a special L-shaped deflection bar is provided, to which is pivoted a vertical arm so arranged that it may be rotated in a vertical plane parallel to the suspended magnet, about a center in the horizontal plane through the suspended magnet.

Cheltenham, Md. June 16, 1905.
Magnetometer No. 29.
Horizontal distance, 21 cm. Vertical distance, 2 cm.

No.	Position of deflecting magnet.		North end.	Horizontal circle readings.		
				A.	B.	Mean.
				° ′ ″	′ ″	° ′ ″
1	East	up	up	53 32 40	32 40	53 32 40
2	East	down	down	53 56 10	56 10	53 56 10
3	East	down	up	43 05 10	05 50	43 05 30
4	East	up	down	43 23 40	23 50	43 23 45
5	West	up	down	54 08 10	08 10	54 08 10
6	West	down	up	54 28 00	28 10	54 28 05
7	West	down	down	42 37 30	37 30	42 37 30
8	West	up	up	43 03 00	03 20	43 03 10

	h. m.		°		
Time of beginning	9 55	Temp.	27.2	$2u_1$ East (1–3)	10 27 10
Time of ending	10 25	Temp.	27.6	$2u_1$ West (6–8)	11 24 55
				Mean	10 56 02
				$\frac{1}{2}u_1$	2 44 00

co log (H=0.20085)	0.6971		
co log tan (I=70° 25′)	9.5511	$2u_2$ East (2–4)	10 32 25
log tan $\frac{1}{2}(u_2-u_1)$	6.6047	$2u_2$ West (5–7)	11 30 40
co log tan $\frac{1}{2}(u_2+u_1)$	1.0172	Mean	11 01 32
log (h=0.0074)	7.8701	$\frac{1}{2}u_2$	2 45 23
" ($M_{27.4}$=697)	2.8432	$\frac{1}{2}(u_2-u_1)$	0 1 23
" (μ=5.17)	0.7133	$\frac{1}{2}(u_2+u_1)$	5 29 23

DISTRIBUTION COEFFICIENTS.

n the deflection formula

$$\frac{H}{M}=\frac{2}{r^3 \sin u}\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)\left(1-\frac{2\mu}{r^3}\right)$$

the distribution coefficients P and Q may be obtained by making deflections at three distances and solving the three resulting equations for the three unknowns $\frac{H}{M}$, P, and Q.

Let $\quad \frac{r_1^3 \sin u_1}{2\left(1-\frac{2\mu}{r_1^3}\right)}=A_1 \qquad \frac{r_2^3 \sin u_2}{2\left(1-\frac{2\mu}{r_2^3}\right)}=A_2 \qquad \frac{r_3^3 \sin u_3}{2\left(1-\frac{2\mu}{r_3^3}\right)}=A_3$

$$\text{Then } P=\frac{A_1r_1^4(r_3^4-r_2^4)+A_2r_2^4(r_1^4-r_3^4)+A_3r_3^4(r_2^4-r_1^4)}{A_1r_1^4(r_2^2-r_3^2)+A_2r_2^4(r_3^2-r_1^2)+A_3r_3^4(r_1^2-r_2^2)}$$

$$Q=\frac{r_1^2r_2^2r_3^2[A_1r_1^2(r_2^2-r_3^2)+A_2r_2^2(r_3^2-r_1^2)+A_3r_3^2(r_1^2-r_2^2)]}{A_1r_1^4(r_2^2-r_3^2)+A_2r_2^4(r_3^2-r_1^2)+A_3r_3^4(r_1^2-r_2^2)}$$

In case the relative lengths of the two magnets are such that Q is nearly zero, the term $\frac{Q}{r^4}$ becomes so small that it may be neglected and the value of P may be computed from deflections at two distances. Using the same notation as above:

$$\frac{A_1}{A_2}=\frac{1+\frac{P}{r_1^2}}{1+\frac{P}{r_2^2}} \quad \text{and} \quad P=\frac{A_1-A_2}{\frac{A_2}{r_1^2}-\frac{A_1}{r_2^2}}=\frac{r_1^2 r_2^2}{A_2 r_2^2-A_1 r_1^2}(A_1-A_2)$$

When P is small it may be computed with sufficient accuracy by the formula

$$P=\log_e 10\frac{r_2^2 r_1^2}{r_2^2-r_1^2}(\log A_1-\log A_2)$$

For deflection distances of 30 cm. and 40 cm. this becomes

$$P=4737\ (\log A_1-\log A_2)$$

It is evident that a small error of observation in the deflections will have a large effect on the accuracy of P, and little dependence can be placed on the result from a single set of observations. It is only from an extended series that a reliable value of the distribution coefficients can be obtained. It is also evident from the form of the factor $\frac{r_2^2 r_1^2}{r_2^2-r_1^2}$ that it is important to have the two deflection distances differ by a considerable amount. Too short a deflection distance is undesirable, however, since any uncertainty in the value of P has too great an effect on the resulting horizontal intensity, and too long a distance reduces the size of the deflection angle so much that a small error of observation has a large effect on the result. For the size of magnets generally used, the distances 30 cm. and 40 cm. are found to be the most satisfactory.

The above formula for P may be used also to find the correction to an adopted value of P required to harmonize subsequent observations. If it is found after a series of observations that the two values of $\log\frac{H}{M}$ computed from deflections at two distances differ systematically, one being greater than the other on the average, the correction to the adopted value of P is given by the formula

$$\varDelta P=\log_e 10\frac{r_2^2 r_1^2}{r_2^2-r_1^2}\left[\log\left(\frac{H}{M}\right)_2-\log\left(\frac{H}{M}\right)_1\right]$$

the quantity in brackets being the mean value for the series.

As already pointed out, approximate values of the distribution coefficients may be computed from the dimensions of the magnets.

If $2l$ and $2l_1$ be the pole distances of the long and short magnets, respectively, then

$$P = 2l^2 - 3l_1^2 \qquad Q = 3l^4 - 15l^2 l_1^2 + \frac{45}{8} l_1^4$$

disregarding the small terms depending on the relative diameters of the magnets. Börgen concluded from his experiments that the pole distance is on the average about 0.805 the length of the magnet, and this conclusion was confirmed in part by the standardization observations at Kew prior to 1904 (Dr. C. Chree, "Law of Action between Magnets." L., E., and D. Phil. Mag., Aug. 1, 1904). From the above formula it will be seen that P should be zero when $l/l_1 = 1.225$, and this ratio has been adopted for the lengths of the two magnets in nearly all of the magnetometers which have been made or remodeled by the Coast and Geodetic Survey. At the same time it has been the practice to regard Q as negligible and to determine the value of P from deflections at two distances in the manner explained above. For seven of the eight magnetometers to which the above ratio applies, the value of P derived in this way is in every case between 0 and -1.0, the average value being -0.54.

To show that this assumption is justifiable, let us examine the results for five magnetometers of the design shown in Figure 3, having magnets 7.375 cm. and 6.025 cm. in length, i. e., in the ratio of 1.224 to 1, and arranged for deflections at distances of 30 cm. and 40 cm. Using Börgen's ratio of pole distance to length of magnet, his formulas give:

$$P = 0 \qquad Q = -350$$

Now suppose a series of deflections at the two distances gives on the average

$$\log A_{30} - \log A_{40} = -0.00020$$

Assuming $Q = 0$ $\qquad P = 4737(\log A_{30} - \log A_{40}) = -0.95$

which is about the upper limit of the values found for this type of magnetometer. On the other hand, if we assume $P = 0$, Q may be computed by the formula:

$$Q = \log_e 10 \frac{r_2^4\, r_1^4}{r_2^4 - r_1^4} (\log A_1 - \log A_2) = -546$$

Finally, we may adopt the value of $Q = -350$ computed from Börgen's formula, and find the value of P which will satisfy the equation:

$$\log\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)_{30} - \log\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right)_{40} = -0.00020$$

As $\frac{P}{r^2}$ and $\frac{Q}{r^4}$ are both very small quantities, we may put

$$\log\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right) = \log\left(1 + \frac{P}{r^2}\right) + \log\left(1 + \frac{Q}{r^4}\right)$$

Hence $$\log\left(1+\frac{P}{r^2}\right)_{30}-\log\left(1+\frac{P}{r^2}\right)_{40}$$

$$=\log\left(1+\frac{Q}{r^4}\right)_{40}-\log\left(1+\frac{Q}{r^4}\right)_{30}-0.00020=-0.00007$$

and $$P=4737(-0.00007)=-0.33$$

The effect of the different values of P and Q on the resulting value of H may be determined by computing the value of $\log\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$ for the three cases.

	$P=-0.95$ $Q=0.$		$P=0$ $Q=-546.$		$P=-0.33$ $Q=-350.$	
	$r=30$cm.	$r=40$cm.	$r=30$cm.	$r=40$cm.	$r=30$cm.	$r=40$cm.
$\frac{P}{r^2}$	−.001056	−.000594	0	0	−.000367	−.000206
$\frac{Q}{r^4}$	0	0	−.000674	−.000213	−.000432	−.000137
$1+\frac{P}{r^2}+\frac{Q}{r^4}$	.998944	.999406	.999326	.999787	.999201	.999657
$\log\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$	9.99954	9.99974	9.99971	9.99991	9.99965	9.99985
Mean	−.00036		−.00019		−.00025	

It will be seen that the difference between the logarithms of the factor for the two distances is in each case the assumed difference between $\log A_{30}$ and $\log A_{40}$, but the mean of the two is greatest for the assumption that $P=0$ and least for the case in which $Q=0$. To determine the effect on a resulting value of H we must take the square root of $\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)$ or divide its logarithm by two. The effect of the above three combinations of distribution coefficients would therefore be to diminish the value of $\log H$ by .00018, .000095 and .000125 respectively. The ratio of the first two values is the number of which the logarithm is .000085 or 1.0002; that is, they differ by only 1 part in 5000. Consequently the error involved in the case of the Coast and Geodetic Survey magnetometers in assuming that Q is negligible does not amount to more than 1 part in 5000 in H and is probably less than that, and is therefore well within the probable error of observation and reduction in field work under favorable conditions.

DEFLECTION DISTANCES.

In a magnetometer with fiber suspension it is impossible to avoid a slight variation in the relative positions of the suspended magnet and the deflection bars and a corresponding variation in the deflection distances. To eliminate the error to which this might give rise, the instruments are made either with two deflection bars, one on either side, or with a single bar having its middle point over the center of the magnetometer. The deflection observations can then be made one-half with magnet east and one-half with magnet west and a small increase of the deflection distances on one side will be balanced by a decrease on the other side.

The distance between corresponding marks on the two bars or on the two halves of the single bar is twice the deflection distance. With the single straight bar, such as is used in the Kew and India Survey pattern magnetometers, this is readily obtained by direct comparison with a standard meter. Experiments at Kew have shown that bars of this type require a slight correction for bending, amounting to an increase of about one part in 10000 in the case of the instruments of the latter type in use by the Coast and Geodetic Survey (Fig. 4).

In the Coast and Geodetic Survey pattern magnetometer (Fig. 3), the two bars are so constructed that their inner ends overlap and are held together by two screws. It is thus possible to fasten them together when not in position on the magnetometer and measure the deflection distances as readily as for a single bar. These bars are very light, since the outer ends are hollow, and it has therefore been considered unnecessary to investigate the question of bending.

DIRECTIONS FOR MAGNETIC OBSERVATIONS ON LAND.

GENERAL DIRECTIONS.

Selection of stations.—The conditions to be satisfied in selecting a magnetic station are freedom from present and probable future local disturbance, whether natural or artificial, combined with convenience of access. A station on suitably situated public property, or property belonging to an educational institution, is to be preferred, as it is less likely to be disturbed or affected by change of the immediate surroundings. Proximity of electric railways, masses of iron or steel, gas or water pipes, buildings of stone or brick should be avoided. A quarter of a mile from the first, 500 feet from the second, 200 feet from the third and fourth may be considered safe distances. The station should be at least 50 feet from a building of any kind. If any doubt arises in the selection of a station as to the existence of local disturbance, two intervisible points 100 yards or more apart should be selected and the magnetic bearing of the line joining them determined at each end. A lack of agreement between the two results would be evidence of local disturbance. Similar tests should then be made in other directions until a satisfactory location is found.

Description of station.—Each point occupied should be described with sufficient detail to render possible its recovery. The description should begin with the general location of the park or field in which the station is situated. This should include the approximate distance and direction from the center of the town or from some point which can be definitely located on a map, so that a check on the latitude and longitude may be available. In case a new station is selected in a town where observations had been made before, the relative positions of the new and old stations should be given, if possible.

There should follow measured distances to the fences or other fixed objects in the immediate vicinity of the station and a description of the manner in which the station is marked. If a meridian line is established, the distance to, and location of, the second stone should be given, the magnetic station being selected so as to form one end of the line. It is desirable also to give a rough sketch showing the relation of the station to surrounding objects, indicating on it the direction of north (which should always be toward the top of the sketch), and the direction of the marks of which the true bearings are determined.

Marks.—These marks should be well-defined objects as nearly in the horizon as practicable and likely to be available for future use.

It is desirable to have the one selected for reference mark in azimuth and declination observations in a southerly direction, so that it may be sighted upon through the opening in the south side of the observing tent. It should be a quarter of a mile or more from the station if possible, so that an error of two or three inches in reoccupying the station or a change of that amount in the position of the marking stone would not materially affect the azimuth of the mark. As an angle of 1′ subtends approximately 1 inch at a distance of 300 feet, the uncertainty at any given distance may be readily computed.

Marking of station.—Every station should be marked in as permanent a manner as conditions will warrant, to assist in its subsequent recovery. Either a natural or artificial stone, a glazed drain pipe, or a post of hard wood can usually be obtained. To avoid being disturbed, the station mark should project little, if any, above the surface of the ground, and it should extend two feet or more into the ground.

Meridian line.—When a meridian line is to be established the azimuth observations must be made with especial care and the computations revised before the stones are set. The line should be not less than 300 feet long (if possible not less than 500 feet), and extra precautions should be taken to secure the marking stones against future disturbance.

Repeat stations.—Where observations are to be made at an old station for the purpose of determining the secular variation, especial effort should be made to reoccupy the precise point at which the earlier observations were made. Any changes in the immediate surroundings should be noted in the description of station. If local conditions have changed to such an extent that a reoccupation is clearly undesirable, then a new station must be established. There may be cases, however, in which it will be best to reoccupy the old station and also establish a new one; e. g., the old station, while not satisfying the requirements of future availability, may still suffice to determine the secular variation since the former observations. When, owing to change in the immediate surroundings or defect of the original description, it is impossible to locate the exact point from the measured distances, the desired result may sometimes be accomplished with the aid of the bearings of prominent objects. Having three well-defined objects which were connected by angular measures at the time of the former occupation, successive trials with the theodolite will serve to locate the spot at which those angular measures are reproduced.

Care of instruments.—Care should be taken to keep the instruments in good adjustment and free from dust. The magnets should be touched with the hands as little as possible and should always be wiped dry with clean chamois or soft tissue paper at the close of observations to prevent them from rusting. They must not be dropped or allowed to touch each other or other iron or steel objects. They should be kept in the instrument box with north end down, packed snugly to avoid jars in transportation. The dip needles should be wiped with tissue paper both before and after observations and the pivots cleaned with pith. In reversing polarities, the bar magnets should be drawn smoothly from center to ends of the needle, as nearly parallel to the axis of the needle as possible. In case the needle projects above the surface of the reversing block the magnets must not bear heavily upon it.

Chronometer.—The utmost care must be exercised in carrying the chronometer. A pocket chronometer requires more careful handling than a watch to secure a constant rate. It must be kept at as uniform a temperature as possible and wound at the same hour each day. It must be protected from jarring or shaking. Past experience indicates that the best results are obtained when it is carried in the trousers watch pocket. Where unusual rough travel is anticipated it is well to compare the chronometer with a well-regulated watch both before and after the journey. At least once a week, and at every station if possible without serious delay, the chronometer correction on standard time should be obtained by means of Western Union or other telegraphic time signals. The chronometer correction and rate are given the sign with which they must be applied. For a chronometer which is fast and gaining they are both negative.

Order of observations.—When a complete instrumental outfit is supplied the observations at a station comprise: Morning and afternoon azimuth, latitude at noon; one set of dip with each of two needles; two sets each of declination, oscillations, and deflections; angular measures between prominent objects. It is desirable that the azimuth observations should be made at nearly equal times (preferably not less than two hours) before and after apparent noon, giving nearly the same altitude of the sun for the morning and afternoon sets. The effect on the azimuth of a small error in latitude is in that way eliminated. Latitude observations should extend not more than 15 minutes before or after apparent noon (maximum altitude of the sun).

As the declination and horizontal intensity are usually changing more rapidly in the morning than in the afternoon, it is preferable to make the magnetometer observations in the afternoon. They should be made in the order: Declination, oscillations, deflections, deflections, oscillations, declination. The second set of deflections and oscillations should be made with magnets inverted, and the horizontal circle should be shifted in azimuth before the second set of declination, in order to bring the readings upon a different part of the circle.

Thermometer.—The same thermometer should be used throughout a set of horizontal intensity observations. It should be placed in the hole in the magnet house during oscillations and near to the *deflecting* magnet during deflections, either in the end of the deflection bar or (in magnetometers of the India Survey pattern) in the box in which the magnet is inclosed. It should be changed from one bar to the other with the magnet. Care must be taken to stop up the hole in the magnet house when the thermometer is not in it. Before beginning observations the thermometer should be examined to see that the mercury column is not broken and that none of the mercury is in the upper recess. A broken column can usually be joined by holding the thermometer in the hand and striking the wrist sharply against the knee, or by attaching it securely to a string and swinging it rapidly in a circle.

Agreement of results.—Before leaving a station the computation should be carried far enough to show that there is nothing essentially wrong with the observations. In good work two consecutive sets of azimuth should agree within one minute and the morning and afternoon sets within two minutes. A greater difference is usually due to lack of adjustment or level of the theodolite, or to a mistake in pointing on the wrong limb of the sun, or in using the wrong line of the diaphragm. In case the morning and afternoon azimuth observations give results differing by more than five minutes, the observations should be repeated. The two sets of declination should agree within two or three minutes when corrected approximately for diurnal variation (see Table IX). The values of log MH for the two sets of oscillations should not differ by more than 0.00100, and the values of log $\frac{H}{M}$ should agree equally well. The corresponding agreement to be expected in the values of T and u can easily be computed for a particular magnetometer and a particular locality.

When the dip results for the two needles differ by more than five minutes in excess of the normal difference of the needles, the observations should be repeated. Thus, if the observations show that on the average needle No. 1 gives a value of dip three minutes greater than No. 2, the observations should be repeated when No. 1 gives a result more than eight minutes greater or two minutes less than No. 2.

The record should be kept with hard pencil (or fountain pen) and entered at once on the proper form (not kept on blank paper and afterwards copied onto the form.) All computations should be made in ink or inked over before the record is sent to the Office. The different sheets will be punched and fastened together in the covers provided (Form 367), arranged in the following order: (1) Description of station, angles connecting the azimuth mark with other prominent objects, and chronometer correction on Standard time (Form 441), (2) latitude observations (Form 267), (3) azimuth observations (Form 266), (4) azimuth computation (Form 269), (5) declination (Form 37), (6) dip (Form 42), (7) oscillations (Form 41), (8) deflections (Form 39).

Abstract.—Before the record is sent to the Office the computation should be completed and a copy made of the results and also of such quantities as would be required to replace the computation in case the record is lost (Form 442). This includes brief description of station, chronometer corrections on Standard time, sun's maximum altitude from latitude observations; mean of chronometer, horizontal and vertical circle readings for each set of azimuth; mean reading of mark and magnet, mean scale reading erect and inverted for each declination set; time of whole number of oscillations and effect of 90° torsion, mean value of $2u$ for each deflection distance, temperature and time of each set of observations; the mean dip with each needle for each half set (before and after reversal of polarities).

Computations.—Five-place logarithms will be used. In the azimuth observations the means of circle readings will be carried to whole seconds, means of times to tenths of a second; similarly in computations. For declination observations, carry mean scale readings to hundredths of a division, balance of computation to tenths of a minute. For oscillations, compute time of one oscillation to four decimal places, mean temperature to tenths of a degree. Compute deflection angles to whole seconds. Dip computations will be carried to tenths of a minute.

To secure the best results, particular attention should be paid to the following points:

Be sure that all articles of iron and steel are removed to a safe distance before beginning magnetic observations.

Be sure that the instrument is level and the levels in adjustment before beginning observations, especially in latitude and azimuth observations.

Be careful to keep the magnets and dip needles dry and clean, especially the pivots of the dip needles.

Handle the chronometer with care at all times.

EQUIPMENT.

Observers engaged exclusively on magnetic work are usually provided with a theodolite magnetometer, a dip circle, a half-second pocket chronometer, a tent, and minor accessories. When magnetic observations are to be made only as opportunity offers in connection with other branches of the field work of the Survey, the equipment is often less complete, either a dip circle with special needles for total intensity observations and a compass attachment for determination of the magnetic declination or simply a compass declinometer for declination alone. In such cases the true meridian is usually known from triangulation or else the instrumental equipment includes a theodolite and timepiece with which the necessary astronomical observations can be made.

In the descriptions of instruments and methods which follow, the erm *alidade* will be used to designate the upper part of the instrument to which are attached the verniers for reading the horizontal circle and of which the motion is controlled by the upper clamp and tangent screw.

LATITUDE FROM OBSERVATIONS OF THE SUN.

For the greater part of the United States only approximate values of the latitude and longitude can be obtained from existing maps. It is usual, therefore, to include latitude observations in the program of work at a magnetic station, in order that the azimuth may be determined from sun observations with the required accuracy. The most convenient method involves the measurement of the sun's altitude at or near apparent noon, using the small theodolite provided for the azimuth observations.

At apparent noon, when the sun is on the meridian

$$\phi=\delta+\zeta$$

ϕ being the latitude of the place, ζ the sun's zenith distance, and δ its declination, south zenith distance and north declination being considered positive for the northern hemisphere. As the sun's declination changes so slowly (the hourly rate of change never amounts to 1′), no appreciable error is introduced by assuming it constant for a series of observations beginning a few minutes before noon and ending a few minutes after noon. The maximum altitude may also be taken as the meridian altitude. The observations are made in the manner shown in the example given below.

The observations should begin about ten minutes before apparent noon and end about ten minutes after noon. Before making the observations, therefore, it is necessary to find the chronometer time of apparent noon, at least approximately, by the method given below. After setting up, leveling, and adjusting the theodolite, as explained later in connection with azimuth observations, the method of observing is as follows:

Form 267.

Observations of Sun for Latitude.

Station, Smyrna Mills, Me.
Theodolite of mag'r No. 20.
Chronometer No. 245.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Temperature, 24° C.

Sun's limb.	V. C.	Chronometer time.	Vertical circle.		
			A.	B.	Mean.
		h. m. s.	° ′ ″	′ ″	° ′ ″
U	R	11 30 04	61 14 00	13 30	61 13 45
L	L	11 31 16	119 23 00	20 00	60 38 30
L	L	11 33 14	119 22 30	19 30	60 39 00
U	R	11 34 38	61 16 30	15 30	61 16 00
U	R	11 36 36	61 17 00	15 30	61 16 15
L	L	11 37 34	119 21 30	19 00	60 39 45
L	L	11 39 32	119 21 30	19 00	60 39 45
U	R	11 40 33	61 17 30	16 00	61 16 45
U	R	11 42 46	61 16 30	15 00	61 15 45
L	L	11 43 30	119 22 30	20 00	60 38 45
			Obs'd max. alt.		60 58 15
			R. & P.		— 27
			h		60 57 48
			ζ		29 02 12
			δ		17 06 18
			ϕ		46 08 30

With the vertical circle to the right of the telescope, point on the sun with its disc bisected by the vertical line of the diaphragm and its *upper* limb tangent to the horizontal line. Record the time of contact as indicated by the chronometer and read and record both verniers, A and B, of the vertical circle. Turn the alidade 180° in azimuth and make another pointing on the sun, but with its lower limb tangent to the horizontal line of the diaphragm, again recording the time and vertical circle reading. As the vertical circle is usually graduated from 0° to 360°, the reading in the first case will be the altitude of the sun's upper limb, but the second reading must be subtracted from 180° to get the altitude of the sun's lower limb. Combining the two gives the altitude of the sun's center and eliminates the vertical collimation error of the theodolite and the index error of the graduation. The observations are continued for 15 or 20 minutes, reversing the circle after the odd pointings, as shown in the above example. The level of the instrument should be examined after the even pointings and corrected if necessary. If the beginning is properly timed, the maximum altitude will occur near the middle of the series. For the field computation the pair of readings is selected which gives the maximum altitude, and their mean, after being corrected for refraction and parallax (Table I), is combined with the sun's declination to get the latitude. The quantities for vertical circle left in the column headed "Mean" are really 180° minus the means of the two vernier readings.

A more accurate value of latitude is obtained by utilizing all the observations by the "method of circum-meridian altitudes," explained in detail in most textbooks on spherical astronomy (see, for example, Chauvenet, Vol. I, page 235). In view of the degree of accuracy required in a magnetic survey, or possible with the small theodolite ordinarily used, many approximations in the method of reduction to the meridian may be made. If h be the sun's meridian altitude, h_0 its altitude (observed) at a time t before (or after) apparent noon, then approximately:

$$h = h_0 + \cos\phi \cos\delta \csc\zeta \frac{2\sin^2 \frac{1}{2}t}{\sin 1''}$$

Let $A = \cos\phi \cos\delta \csc\zeta$ and $m = \frac{2\sin^2 \frac{1}{2}t}{\sin 1''}$

Then $h = h_0 + Am$ and $\phi = \delta + \zeta = \delta + 90° - h$

Table III gives the values of m for different values of t and Table IV gives the values of A for different values of ϕ and δ. It will be seen that A increases as the sun's zenith distance decreases, and the method is therefore not well adapted for observations where the sun crosses the meridian near the zenith. The following computation of the set of observations given above will best illustrate the method.

Form 268.

COMPUTATION OF LATITUDE FROM CIRCUM-MERIDIAN ALTITUDES OF SUN.

Station, Smyrna Mills, Me. Date, August 5, 1910.

	h. m. s.
Chron. correction on L. M. T.	+ 27 20
Local mean time of app. noon	12 05 53
Chron. time of apparent noon	11 38 33

t	m	A	A m	Reduced h of sun's limb.	Reduced h of ☉
m. s.	"		"	° ′ ″	° ′ ″
−8 29	141	1.36	192	61 16 57	60 58 54
−7 17	104		141	60 40 51	
−5 19	56		76	60 40 16	60 58 28
−3 55	30		41	61 16 41	
−1 57	8		11	61 16 26	60 58 07
−0 59	2		3	60 39 48	
+0 59	2		3	60 39 48	60 58 22
+2 00	8		11	61 16 56	
+4 13	35		48	61 16 33	60 58 12
+4 57	48		65	60 39 50	
				Mean	60 58 25
				R. & P.	− 27
				h	60 57 58
				ζ	29 02 02
				δ	17 06 19
				ϕ	46 08 21

In order to obtain the values of t, the time of observation before or after apparent noon, it is necessary to find the chronometer time of apparent noon. The chronometer correction on local mean time is usually obtained from the observations of the sun for azimuth and time, as will be seen later, but it may be obtained also from an approximate value of the longitude and the chronometer correction on standard time, as follows: For Smyrna Mills, Me., suppose the approximate longitude, 68° 08′.5, or $4^h\ 32^m\ 34^s$, to be obtained from a map. By comparison with Western Union telegraphic time signals on August 4, 1910, chronometer No. 245 was $6^s.0$ fast on 75th meridian mean time. On August 5 it was $5^s.8$ fast, showing a loss of $0^s.2$ per day.

	m.	s.
At noon August 5 chronometer No. 245 correction on 75th mer. m. t.		−6
Smyrna Mills east of 75th meridian	27	26
Therefore chronometer No. 245 correction on local mean time	+27	20

To find the local mean time of apparent noon, we must know the difference between mean time and apparent time; that is, the equation of time, E. This may be found in any ephemeris. On page 128 of the American Ephemeris for 1910, the value of E at Greenwich apparent noon on August 5 is found to be $5^m\ 53^s.7$ and decreasing at the rate of $0^s.23$ per hour. Hence, for Smyrna Mills apparent noon, which occurs $4^h\ 32^m$ later, the value of E would be $5^m\ 53^s.7 - 1^s.0 = 5^m\ 52^s.7$. This is the amount which must be added to apparent time in order to obtain mean time. As the equation of time for apparent noon never differs from the equation of time for mean noon by as much as $0^s.2$, the latter may be used when the sun's ephemeris for apparent noon is not available. The apparent time of apparent noon is always $12^h\ 00^m\ 00^s$, hence the mean time of apparent noon at Smyrna Mills on August 5, 1910, was $12^h\ 05^m\ 53^s$. The chronometer was found to be $27^m\ 20^s$ slow of local mean time. Hence the chronometer time of apparent noon was $11^h\ 38^m\ 33^s$. Expressed analytically: *Chronometer time of apparent noon = mean time of apparent noon − chronometer correction,* bearing in mind that the correction is considered positive when the chronometer is slow and negative when it is fast. By subtracting the chronometer time of apparent noon from the chronometer time of each observation, the corresponding value of t is found. This is the argument required for obtaining m from Table III.

For obtaining the values of A from Table IV only approximate values of ϕ and δ are required, but as the value of δ is needed later it is just as well to compute it at this point.

	h.	m.	s.
Chronometer time of apparent noon	11	38	33
Chronometer correction on 75th meridian mean time	−		06
75th meridian time slow on Greenwich mean time	+5	00	00
Greenwich mean time of local apparent noon	4	38	27

This is the Greenwich mean time for which the sun's declination is required. By referring to page 129 of the American Ephemeris for 1910 it will be found that the sun's declination at Greenwich mean noon on August 5 was 17° 09′ 25″.8 N., and decreasing at the rate of 40″.19 per hour. This rate is not uniform, however, the value for noon of the 6th being 40″.88. As the declination at 4^h 38^m 27^s, or $4^h.64$ after noon is required, we must find the average hourly change for that interval, or, what is approximately the same thing, the change for the middle of the interval, or $2^h.32$ after noon. As the hourly change increased 0″.69 in 24 hours, it increased about $\frac{1}{10}$ of of that amount, or 0″.07 in 2.3 hours, and the desired average value for the interval is therefore 40″.19 + 0″.07 = 40″.26.

	° ′ ″
Sun's declination at Greenwich mean noon Aug. 5	17 09 25.8 N.
Change for $4^h.64 = 4.64 \times 40''.26 = 186''.8$	−3 06.8
Sun's declination at 4^h 38^m 27^s after noon	17 06 19.0 N.

In practice it is sufficient to carry the hourly change to tenths of seconds only, and the allowance for second differences may then be made by inspection. In fact, in most cases second differences might be neglected entirely, as the maximum error would be only 3″.5 in assuming that the tabular value of hourly change is uniform for 12 hours before and 12 hours after the noon to which it refers.

The product Am is the difference between the altitude of the sun at noon and at the time t before or after noon, and must therefore be added to the observed altitude in order to get the corresponding meridian altitude. The altitude of the sun's center is found by combining an altitude of the upper limb with one of the lower limb. The mean of the different results is treated as the maximum altitude was in the approximate field computation.

LATITUDE FROM OBSERVATIONS OF POLARIS.

If desired the latitude may be readily determined by observing the altitude of the pole star, when the longitude and local mean time are known approximately, using the formula:

$$\phi = h - p \cos t + \frac{1}{2} p^2 \sin^2 t \sin 1'' \tan h$$

t being the hour-angle of the star, p its polar distance, and h the observed altitude corrected for refraction. The refraction may be obtained from Table I if the tabular quantities are increased by 8″.8 cos h, the amount of the solar parallax. When observations are made at upper or lower culmination, the formula becomes

$$\phi = h \mp p$$

The right ascension and declination of Polaris for each day of the year are given in the American Ephemeris. There also will be found on the last page of the book a table giving the altitude of the star above or below the pole at any hour-angle, computed for latitude 45° and the mean declination of the star for the year.

DETERMINATION OF THE TRUE MERIDIAN AND LOCAL MEAN TIME BY MEANS OF OBSERVATIONS OF THE SUN.

The following method is the one usually employed to determine the true meridian in connection with the magnetic observations of the Coast and Geodetic Survey. It is more convenient than others in that it may be employed during daylight when the magnetic observations are in progress. In connection with the time signals sent out by telegraph from astronomical observatories it furnishes the means also of determining approximately the longitude of the place of observation. It requires a theodolite with a vertical circle and prismatic eyepiece for observing the sun, and a well-regulated timepiece. The observations at a place usually consist of four independent sets of observations, two in the morning and two in the afternoon, each set comprising four pointings on the sun and two pointings on a reference mark, symmetrically arranged as in the example given below. For each pointing on the sun the time is noted, and the horizontal and vertical circles are both read. For the best results the observations should be made not less than two hours from apparent noon.

ADJUSTMENT OF THE THEODOLITE.

Before beginning observations it is necessary to see that the theodolite is in good adjustment, especially as regards the levels.

To adjust the levels.—After mounting the theodolite on the tripod, set up the instrument over the station mark with the tripod head approximately level and the legs planted firmly in the ground or resting on suitable stubs. Most small theodolites are provided with a quick centering device, by means of which the accurate setting over the station mark is made after the tripod has been fixed in position. Turn the alidade until one of the levels is parallel to the line joining two of the leveling screws. Bring the level bubble to the center of the vial by means of the leveling screws. Bring the bubble of the second level to the center of its vial by means of the third leveling screw (by the other pair of leveling screws, if there are four). If necessary, repeat the operation until both bubbles are in the center. Then turn the alidade 180° in azimuth. If the levels are out of adjustment, the bubbles will no longer be in the center of the vials. Correct one half of the defect by means of the adjusting screws of the levels and the other half by means of the leveling screws. Return the alidade to its original position and repeat the operation if necessary. When the adjustment has been completed the instrument will be level and the level bubbles will be in the center of the vials no matter in what direction the telescope is pointing.

To insert new cross wires.—The cross wires of a telescope are attached to a metal ring which is held in position near the eyepiece by four capstan screws. They may be spider threads (obtained from a cocoon, not from a web), or fine platinum wire, or, more commonly, lines etched on a thin piece of glass, called a diaphragm, which is fastened to the ring by shellac. An extra diaphragm and a small bottle of shellac should be kept with the instrument so that the observer may insert a new diaphragm should he be so unfortunate as to break the old one. To do this the eyepiece is removed, the ring taken out, and the remains of the old diaphragm and shellac cleaned off. The ring is then laid on a piece of white paper and the new diaphragm placed in the position indicated by lines on the ring and fastened by shellac around the edges. The ring is then put back in the telescope tube, and adjusted in position by means of the capstan screws as explained later.

To adjust the eyepiece.—Point the telescope to the sky and move the eyepiece in or out until the cross wires appear sharp and distinct. Then turn the telescope to a distant object and move the object glass in or out until the object appears sharply defined. If the adjustment has been properly made there will be no apparent motion of the object when the eye is moved from one side of the eyepiece to the other. If the vertical cross wire is perpendicular to the horizontal axis of the theodolite, an object which has been bisected by one part of the wire will continue to be bisected throughout the length of the wire when the telescope is revolved about its horizontal axis. If this is not the case the capstan screws should be loosened and the ring carrying the cross wires rotated slightly about the optical axis. In the field the verticality of this cross wire may be tested by pointing on the vertical edge of a house. At the same time the horizontality of the transverse axis of the telescope may be tested by turning the telescope in altitude and seeing whether the edge of the house remains bisected for a considerable change in altitude.

To adjust the vertical cross wire for collimation.—Point at a well-defined distant object. Turn the alidade 180° in azimuth and reverse the telescope and point on the object again. The amount by which the difference of the two circle readings differs from 180° is twice the error of collimation and may be corrected by moving laterally the ring carrying the cross wires, by means of the capstan screws on the sides of the telescope tube. When the telescope is mounted eccentrically, as it is in some magnetometers, allowance must be made for that fact in adjusting for collimation. Two marks must be provided which are twice as far from each other as the optical axis of the telescope is from the vertical axis of the instrument.

This adjustment is usually attended to by the mechanician before the instrument is sent from the office and rarely needs to be repeated in the field unless it becomes necessary to insert new cross wires, since the observations are so arranged as to eliminate small errors of collimation.

To adjust the vertical circle to read zero when the telescope is level.—While the observations are usually so arranged as to eliminate the effect of index error of the vertical circle and vertical collimation error of the telescope, it is desirable to keep that error small so that a setting on the wrong limb of the sun or an error in reading the circle may be more readily discovered. This adjustment is made by means of a slow-motion screw which operates on an arm of the frame carrying the verniers by which the vertical circle is read. Bisect a distant object with the horizontal cross wire and read the vertical circle. Turn the alidade 180° in azimuth, invert the telescope, and again point on the object and read the vertical circle. If the sum of the two readings differs from 180°, correct for half the difference by means of the slow-motion screw which moves the verniers. When this adjustment has been made, the level attached to the vernier frame may be adjusted also. In some theodolites the vertical circle is not attached rigidly to the telescope, but is held by friction or by a clamp. In making the above adjustment for an instrument of that class, a first approximation is obtained by shifting the position of the graduated circle, and then the process is completed by moving the verniers.

OBSERVATIONS.

Having leveled and adjusted the theodolite and selected a suitable azimuth mark, a well-defined object nearly in the horizon and more than 100 yards distant, the azimuth observations are made in the following order, as shown in the sample set given below.

Point on the mark with vertical circle to the right of the telescope (V. C. R.) and read the horizontal circle, verniers *A* and *B*. Reverse the circle, invert the telescope and point on the mark again, this time with vertical circle left (V. C. L.). Place the colored glass in position on the eyepiece and point on the sun with vertical circle left, bringing the horizontal and vertical cross wires tangent to the sun's disc. At the moment when both cross wires are tangent note the time by the chronometer. If an appreciable interval is required to look from the eyepiece to the face of the chronometer, the observer should count the half-seconds which elapse and deduct the amount from the actual chronometer reading. The horizontal and vertical circles are then read and recorded. A second pointing on the sun follows, using the same limbs as before. The alidade is then turned 180° and the telescope inverted and two more pointings are made, but with the cross wires tangent to the limbs of the sun opposite to those used before reversal. This completes a set of observations. A second set usually follows immediately, but with the order of the pointings reversed, ending up with two pointings on the mark. Between the two sets the instrument should be releveled if necessary.

To avoid the necessity of moving both tangent screws in making a setting on the sun, it is convenient to clamp the circles with one cross wire slightly in advance of the limb and then wait until the limb moves up to it, at the same time keeping the other cross wire tangent by means of the tangent screw. As the wires are seen more distinctly when brightly illuminated, the limbs to be observed should be so selected that one wire may cross the sun's disc until the moment of tangency is reached. The observer must be sure to point on opposite limbs in the two halves of a set, so that the mean of the four readings will refer to the sun's center. If he should make the mistake of pointing on the wrong limb, the reading must be corrected for the sun's diameter. For a vertical circle reading the correction is the diameter, which may be obtained from an ephemeris of the sun or with sufficient accuracy from the second column of Table II. For a horizontal circle reading, the sun's diameter must be divided by the cosine of the sun's altitude in order to get the desired correction. The values for altitudes from 10° to 70° are given in Table II.

Form 266.

Observations of Sun for Azimuth and Time.

Station, Smyrna Mills, Me.
Theodolite of mag'r No. 20.
Mark, Flagpole on school building.
Chronometer, 245.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Temperature, 20° C.

Sun's limb.	V. C.	Chronometer time.	Horizontal circle.			Vertical circle.		
			A.	B.	Mean.	A.	B.	Mean.
			° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
	L	Mark	124 43 40	43 50	124 43 45			
	R		304 43 40	43 40	304 43 40			
					124 43 42			
		h. m. s.						
☉	R	8 25 54	155 12 30	12 50	155 12 40	41 10 30	11 30	41 11 00
☉	R	27 56	155 40 40	41 00	155 40 50	41 31 00	31 30	41 31 15
☉	L	30 03	337 00 10	00 20	337 00 15	138 47 00	45 30	41 13 45
☉	L	32 06	337 28 20	28 30	337 28 25	138 27 00	25 30	41 33 45
		8 28 59.8			336 20 32			41 22 26
								— 57
☉	L	8 33 45	337 51 00	51 20	337 51 10	138 10 30	09 00	41 50 15
☉	L	35 59	338 24 30	24 50	338 24 40	137 48 30	47 00	42 12 15
(	R	38 20	158 10 00	10 20	158 10 10	43 10 30	11 30	43 11 00
☉	R	40 37	158 41 50	42 10	158 42 00	43 31 30	32 30	43 32 00
		8 37 10.2			338 17 00			42 41 23
								— 54
	R	Mark	304 43 40	43 50	304 43 45			
	L		124 43 20	43 40	124 43 30			
					124 43 38			

Form 266.

Observations of Sun for Azimuth and Time.

Station, Smyrna Mills, Me.
Theodolite of mag'r No. 20.
Mark, Flagpole on school building.
Chronometer, 245.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Temperature, 21° C.

Sun's limb.	V. C	Chronometer time.	Horizontal circle.			Vertical circle.		
			A.	B.	Mean.	A.	B.	Mean.
			° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
	R	Mark	280 45 00	45 20	280 45 10			
	L		100 45 20	45 40	100 45 30			
					280 45 20			
		h. m. s.						
☉⌟	L	3 12 38	96 35 50	36 20	96 36 05	143 03 00	00 00	36 58 30
☉⌟	L	14 38	97 01 20	01 50	97 01 35	143 23 00	20 00	36 38 30
⌜☉	R	16 44	278 04 10	04 30	278 04 20	36 53 30	53 00	36 53 15
⌜☉	R	18 46	278 29 00	29 20	278 29 10	36 33 00	32 00	36 32 30
		3 15 41.5			277 32 48			36 45 41
								— 1 08
⌜☉	R	3 20 12	278 47 20	47 40	278 47 30	36 18 00	17 30	36 17 45
⌜☉	R	22 12	279 12 00	12 20	279 12 10	35 58 00	57 30	35 57 45
☉⌟	L	23 50	98 57 40	58 10	98 57 55	144 56 30	53 30	35 05 00
☉⌟	L	25 50	99 22 40	23 10	99 22 55	145 17 00	14 00	34 44 30
		3 23 01.0			279 05 08			35 31 15
								— 1 11
	L	Mark	100 45 20	45 40	100 45 30			
	R		280 45 20	45 30	280 45 25			
					280 45 28			

The chronometer and circle readings for the four pointings of a set are combined to get mean values for the subsequent computation. When the vertical circle is graduated from zero to 360°, the readings with vertical circle right give the apparent altitude of one limb of the sun, while those with vertical circle left must be subtracted from 180° to get the apparent altitude of the other limb. The mean of the four pointings gives the apparent altitude of the sun's center. This must be corrected for refraction and parallax to get the true altitude. The value of this correction is given in Table I for different temperatures and altitudes for average conditions. The change in refraction with change in barometric pressure need not be taken into consideration. The correction for refraction is so large and uncertain near the horizon that observations of the sun should be avoided when its altitude is less than 10°.

It is important to test the accuracy of the observations as soon as they have been completed, so that additional sets may be made if

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necessary. This may be done by comparing the mean of the first and fourth pointings of a set with the mean of the second and third, or by comparing the rate of change in the altitude and azimuth of the sun between the first and second pointings, the third and fourth, fourth and fifth, fifth and sixth, and seventh and eighth. For the period of 15 or 20 minutes required for two sets of observations the rate of motion of the sun does not change much.

COMPUTATION.

For the computation of the azimuth of the sun and the local mean time from observations made in the above manner, use is made of the following formulas, the derivation of which has been explained in the first part of this publication.

$$\operatorname{ctn}^2 \tfrac{1}{2}A = \sec s \sec (s-p) \sin (s-h) \sin (s-\phi)$$

$$\tan \tfrac{1}{2}t = \sin (s-h) \sec (s-p) \tan \tfrac{1}{2}A$$

A = azimuth of sun, east of south in the morning, west of south in the afternoon.

ϕ = latitude of the place.

h = altitude of the sun corrected for refraction and parallax.

p = polar distance of the sun at the time of observation.

$s = \tfrac{1}{2}(h+\phi+p)$.

t = the hour angle of the sun, or apparent time of observation, expressed in arc.

The form of computation is shown in the following example, for the sets of observations at Smyrna Mills, Me., given above.

Form 269.

COMPUTATION OF AZIMUTH AND LONGITUDE.

Station, Smyrna Mills, Me.

Date.	Aug. 5.	Aug. 5.	Aug. 5.	Aug. 5.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
h	41 21 29	42 40 29	36 44 33	35 30 04
ϕ	46 08 21	46 08 21	46 08 21	46 08 21
p	72 51 34	72 51 39	72 56 08	72 56 12
$2s$	160 21 24	161 40 29	155 49 02	154 34 37
s	80 10 42	80 50 14	77 54 31	77 17 18
$s-p$	7 19 08	7 58 35	4 58 23	4 21 06
$s-h$	38 49 13	38 09 45	41 09 58	41 47 14
$s-\phi$	34 02 21	34 41 53	31 46 10	31 08 57
log sec s	0.76807	0.79795	0.67887	0.65749
" sec $(s-p)$	0.00355	0.00422	0.00164	0.00125
" sin $(s-h)$	9.79718	9.79091	9.81839	9.82371
" sin $(s-\phi)$	9.74800	9.75530	9.72140	9.71372
" ctn² ½ A	0.31680	0.34838	0.22030	0.19617
" ctn ½ A	0.15840	0.17419	0.11015	0.09808
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
A from South	69 33 04	67 36 43	75 37 17	77 10 09
Circle reads	336 20 32	338 17 00	277 32 48	279 05 08
S. Mer. "	45 53 36	45 53 43	201 55 31	201 54 59
Mark "	124 43 42	124 43 38	280 45 20	280 45 28
Azimuth of Mark	78 50 06	78 49 55	78 49 49	78 50 29
Mean	78 50 05			
log sec $(s-p)$ sin $(s-h)$	9.80073	9.79513	9.82003	9.82496
" tan ½ t	9.64233	9.62094	9.70988	9.72688
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
t in arc	47 23 24	45 20 52	54 17 25	56 07 55
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
t	−3 09 33.6	−3 01 23.5	3 37 09.7	3 44 31.7
E	+ 5 53.4	+ 5 53.3	+ 5 51.8	+ 5 51.8
Local M. T.	8 56 19.8	9 04 29.8	3 43 01.5	3 50 23.5
Chron. time	8 28 59.8	8 37 10.2	3 15 41.5	3 23 01.0
Δt on L. M. T.	+ 27 20.0	+ 27 19.6	+ 27 20.0	+ 27 22.5
Δt on 75 M. T.	— 5.8	— 5.8	— 5.8	— 5.8
$\Delta\lambda$	−27 25.8	−27 25.4	−27 25.8	−27 28.3
Mean	−27 26.3=	− 6° 51′.6	λ=68° 08′.4	

The different steps of the computation are most conveniently made in the following order:

Enter the corrected altitude, mean readings of the horizontal circle for the pointings on the sun and on the mark, and the chronometer time for each set of observations in their proper places. Enter the value of latitude obtained from the latitude observations or other source. Compute the chronometer correction on standard time for the time of each set of observations from the comparisons with telegraphic time signals. Unless the chronometer has a large rate its correction may be taken the same for two contiguous sets of observations. Compute the Greenwich mean time of observation for each set, and find from the American Ephemeris, or H. O. Publication No. 118, the sun's polar distance and the equation of time for that time in the manner explained in connection with the computation of latitude from circum-meridian altitudes. The succeeding steps require little explanation. As the horizontal circles of theodolites are with few exceptions graduated clockwise, and as the sun is east of south in the morning and west of south in the afternoon, it follows that in order to find the horizontal circle reading of the south point, the azimuth of the sun must be added to the circle reading of the sun for the morning observations and subtracted from it for the afternoon observations. The horizontal circle reading of the south point subtracted from the mark reading gives the azimuth of the mark, counted from south around by west from 0 to 360°.

For the computation of t, the logarithms of sec $(s-p)$ and sin $(s-h)$ are found in the azimuth computation and their sum can be written down in its proper place. From that must be subtracted log ctn $\frac{1}{2}A$ to find log tan $\frac{1}{2}t$. The corresponding value of t is the time before or after apparent noon. If in the case of the morning observations ctn $\frac{1}{2}t_1$ be substituted for tan $\frac{1}{2}t$, t_1 will be counted from midnight. The difference between the chronometer correction on local mean time and the correction on standard time is the difference in longitude between the standard meridian and the place of observation.

The angular measures connecting selected prominent objects are conveniently made in connection with the mark readings at the close of the azimuth observations. The various marks should be pointed on successively with vertical circle left and then in the reverse order with vertical circle right. They should be well-defined objects not liable to be confused with similar ones near by. The edge of a chimney, for example, is not a desirable mark, as there is always danger of confusing the edges as they appear to the naked eye with their reversed position as seen through the telescope.

DETERMINATION OF THE TRUE MERIDIAN FROM OBSERVATIONS OF POLARIS.

The true meridian may also be determined by measuring the angle between Polaris and a reference mark, when the local mean time is known. The most convenient time for observing is just after sunset, when the mark does not require illumination. The azimuth of the star from the north is computed by means of the formula

$$\tan A = \frac{-\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

in which t is the hour angle of the star before or after upper culmination and δ is its declination. For the purposes of magnetic work it is sufficient to know the local mean time within one or two minutes. At elongation the change in the azimuth of Polaris is inappreciable for a considerable interval and even a less accurate knowledge of the time will suffice. When the local time is not known the time of culmination of Polaris may be determined with sufficient accuracy from a knowledge of its position with relation to ζ Ursæ Majoris or δ Cassiopeiæ.

A detailed explanation of these methods of determining the true meridian, together with tables to facilitate their use, will be found in "Principal Facts of the Earth's Magnetism," pages 79–91.

DETERMINATION OF THE MAGNETIC DECLINATION.

A. WITH A MAGNETOMETER.

The determination of the magnetic declination consists of two operations; first, the determination of the true meridian as explained in the preceding section, and second the determination of the magnetic meridian, using either a magnetometer, a compass declinometer, or the compass attachment of a dip circle.

Coast and Geodetic Survey pattern magnetometer.—Most of the magnetometers in use in the field work of the Coast and Geodetic Survey are similar in design to the one shown in Figure 3. It is usually referred to as a theodolite magnetometer since it comprises a theodolite and a magnetometer arranged for mounting on the same base. It is light, compact, of simple construction, and easily handled and is therefore especially suited to the field work of a magnetic survey. The horizontal circle is 5 inches in diameter graduated to 20′ and read by two verniers to 20″. The magnets are hollow, octagonal, 1.1 cm. between opposite faces. The lengths of the two magnets (7.4 and 6.0 cm.) are such as to make the first distribution coefficient (P) nearly zero. The observer faces south when making observations of the suspended magnet. In the south end of each magnet is a graduated scale and in the north end a collimating lens so arranged that when the reading telescope is focused on a distant object the graduated scale will be in focus also. The magnet is supported in a brass stirrup consisting of three parallel connected rings joined to a shank about 2.5 cm. long. This long shank prevents any appreciable change of level of the magnet for a considerable change of vertical force. A short pin in the center of the stirrup engages a groove about the center of the magnet. With the octagonal form of magnet the scale is easily placed horizontal in either the erect or inverted positions. When not in use the stirrup is attached to a hook under the roof of the magnet house to prevent breaking or twisting of the fiber.

Silk fiber suspension is used, two strands usually being sufficient to support the magnets without danger of breaking. The upper ends of the fibers are held by a clamp, with a suitable arrangement of screw and nut or rack and pinion for regulating the height of the suspended magnet.

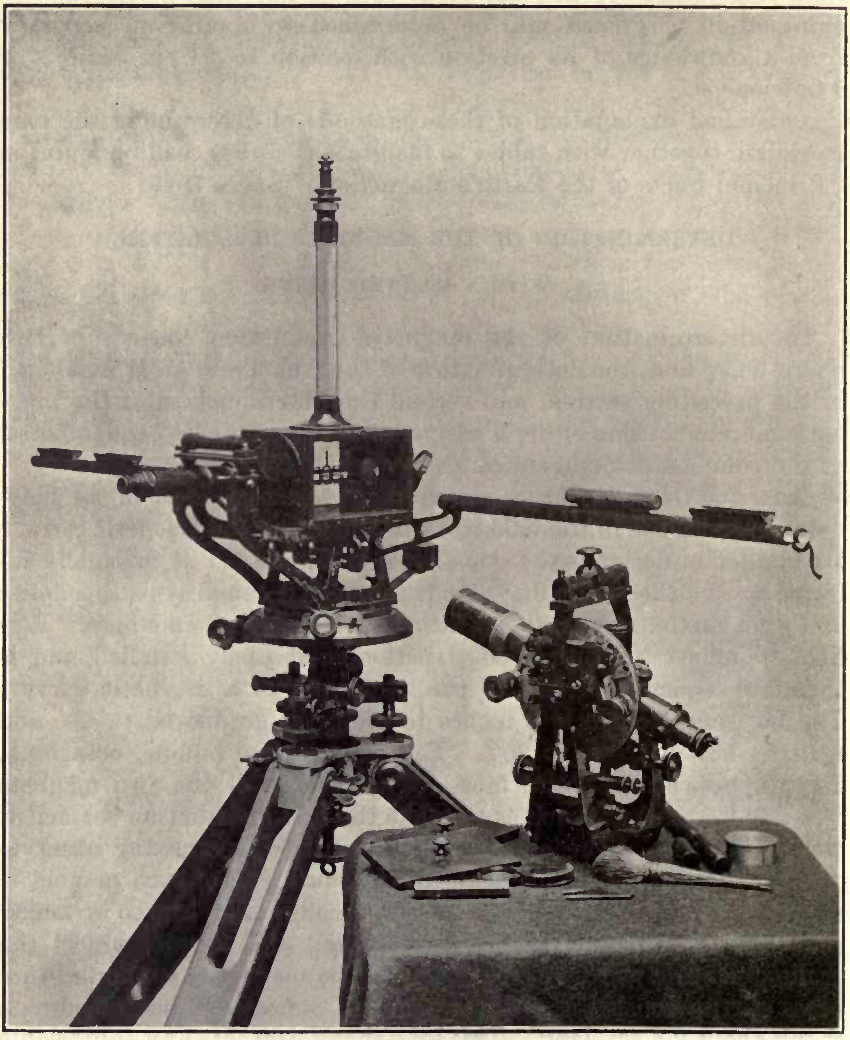

FIG. 3.—Coast and Geodetic Survey pattern magnetometer.

The end of the reading telescope is connected with one end of the wooden magnet house by a hood of dark cloth, so that no glass comes between the objective and the magnet. Light to illuminate the scale of the magnet is admitted through a hole in the other end of the magnet house. This hole is closed by a glass window, which is opened when pointings are to be made on the mark in declination observations, in order to avoid the distortion likely to be caused by irregular refraction of the glass.

The deflection bars used in the horizontal intensity observations are of such shape that the deflecting magnet when in position on the bar is on a level with the optical axis of the reading telescope and at right angles with it and consequently with the suspended magnet also. The bars are not graduated, but on each there are two troughs for supporting the deflecting magnet. In the middle of each trough is a short pin which fits into the groove around the magnet and thus insures its proper setting. The pins are approximately 30 and 40 cm. from the center of the magnet house. In Figure 3 the long magnet is in position on the deflection bar, and the wooden sides of the magnet house have been removed to show the suspended short magnet. The theodolite shown at the right of the picture is easily mounted in place of the magnetometer when azimuth or latitude observations are to be made.

In order to minimize the change in torsion with change in atmospheric conditions, the silk fibers should be well soaked in glycerin before they are used. Extra fibers should be kept in soak in a bottle of glycerin, to be ready for use in case of breakage. A convenient way to insert new fibers is as follows: Draw the fibers through the fingers several times to remove superfluous glycerin and undesirable twists. Fasten one end to the eye of the stirrup with a small loop. Draw the fibers even and fasten a small piece of wax or other weight to the loose ends. Remove the torsion head from the suspension tube, turn the magnet house upside down, and drop the weighted ends through the tube. The wax may then be removed and the ends fastened to the torsion head at the proper distance from the stirrup, care being taken to have the two fibers of the same length. When the torsion head is at its lowest position the stirrup should be about half an inch above the floor of the magnet house. Especial care must be taken to leave no loose ends which might touch the magnet house or the inside of the suspension tube.

The determination of the magnetic meridian with this type of magnetometer is made as follows: Mount the magnetometer and level carefully by means of the striding level provided for the reading telescope (shown in position in the picture). Turn the alidade until the telescope points approximately magnetic south. Place the thermometer in the hole in the roof of the magnet house, suspend the torsion weight (a solid brass cylinder of about the same mass as the long magnet), and replace the wooden sides of the magnet house by those of glass. Bring the torsion weight to rest and then watch its vibration under the influence of the twist of the suspension fibers. By successive trials turn the torsion head at the top of the suspension tube until the weight comes to rest in a position parallel to the optical axis of the telescope, or until its arc of vibration, reduced to a small amount, is bisected by that line. The suspension is then free from twist—that is, there is no tendency to turn a suspended weight out of the vertical plane through the optical axis of the telescope—and the reading of the torsion head indicates the line of detorsion. With a silk fiber suspension just strong enough to support the magnet, the effect of 90° of torsion seldom amounts to as much as 5′ and an error of 10° in the determination of the line of detorsion would therefore affect the resulting declination by not more than 0′.5.

Open the glass window in the end of the magnet house and point upon the object used as a reference mark in the azimuth observations, lowering the torsion weight below the line of sight. Read both verniers and enter the readings in the proper place in the record.

Close the window, turn the alidade until the telescope again points approximately magnetic south, remove the torsion weight and suspend in its place the long magnet with its scale erect, being careful to slacken the fibers as little as possible. Raise the magnet to the level of the reading telescope, quiet its vibration as much as possible, and replace the wooden sides of the magnet house. Adjust the mirror so that it reflects the light onto the scale of the magnet.

Check the vibration of the magnet until the arc is reduced to one or two divisions of the scale. This may be done by means of a screw-driver, adjusting pin, or other small piece of steel, holding it a short distance from the end of the magnet, first on one side and then on the other as the magnet swings back and forth. It will often be found that an adjusting pin has become slightly magnetized. In such cases the alternate attraction and repulsion may be produced by turning the pin end for end.

Turn the alidade until the division of the scale corresponding to the magnetic axis swings by about equal amounts to the right and left of the vertical wire of the reading telescope, and clamp the horizontal circle. If the scale reading of the axis is not known approximately from previous observation, the middle division of the scale will be used. *This setting of the horizontal circle is not to be changed until the time comes to point on the mark again.*

Read the scale when the magnet comes to rest momentarily at the extremes of its swing. When the scale is not numbered it is assumed to be erect when the longer divisions project upward and the readings are then considered as increasing from left to right. The "left" reading is the one when the left end of the scale approaches nearest to the vertical wire of the reading telescope, and is therefore less than the "right" reading for magnet erect. After an interval of a minute read the scale again.

Turn the magnet upside down in the stirrup, so that the scale appears inverted, reduce the arc of vibration, and make four readings of the scale at intervals of one minute. The zero of the scale is now to the right, and the "left" reading will be greater than the "right."

Return the magnet to the erect position and make two more scale readings.

Read the horizontal circle to be sure that it has not been disturbed accidently; remove the magnet and complete the set by pointing on the reference mark. When horizontal intensity observations are to follow immediately, as is usually the case, it is more convenient to make the first set of oscillations before removing the magnet and repeating the pointing on the mark.

The mean of the erect and inverted readings gives the division of the scale which corresponds to the position of the magnetic axis. When the telescope is pointed on that division, it is in the plane of the magnetic meridian. For any other scale reading the reading of the horizontal circle must be corrected by the angular value of the portion of the scale included between the observed scale reading and the scale reading of the axis. With magnet erect the zero of the graduation is at the apparent left and increasing scale readings correspond to decreasing circle readings. Under ordinary conditions the scale reading of the axis of a magnet will remain very nearly constant for a long time. If it shows much variation from station to station, the magnet should be examined carefully to make sure that the scale glass and its mounting are not loose.

The angular value of one division of the scale is readily determined by pointing successively on every fifth or every tenth division and reading the horizontal circle in each case, then repeating the operations in the reverse order, so as to eliminate gradual change of declination during the observations, as shown in the following example, the order of observations being indicated by the figures in the second and fourth column:

Scale Value of Magnet 11L of Magnetometer No. 11.

Scale reading.	First set.		Second set.		Mean.	Value of 30 divisions.
		° ′ ″		° ′ ″	° ′ ″	
0	1	146 46 45	12	146 46 15	146 46 30	(0–30)
10	2	146 11 15	11	146 10 30	146 10 52	(10–40)
20	3	145 33 30	10	145 32 30	145 33 00	(20–50)
						° ′ ″
30	4	144 57 45	9	144 57 15	144 57 30	1 49 00
40	5	144 20 45	8	144 20 00	144 20 22	1 50 30
50	6	143 42 30	7	143 42 30	143 42 30	1 50 30
					30 divisions	1 50 00
					1 division	3′.67

The accompanying example showing the form of record and computation needs little explanation.

Form 37.

MAGNETIC DECLINATION.

Station, Smyrna Mills, Me. | Date, Friday, August 5, 1910.
Magnetometer No. 20. | Observer, H. E. McComb.
Mark, Flagpole on school building.
Magnet, 20L. | Line of detorsion, 80°.

Chron. time.	Scale.	Scale readings. Left.	Right.	Mean.	Horizontal circle readings.		Mark.	Magnet.
h. m.		d.	d.	d.			° ′ ″	° ′ ″
9 23	E	28.8	30.2	29.50	Before	A	325 38 40	226 46 20
9 24	E	28.9	30.3	29.60		B	145 39 00	46 46 40
9 26	I	28.7	25.6	27.15	After	A	325 38 40	226 46 20
9 27	I	28.7	25.7	27.20		B	145 39 00	46 46 40
9 28	I	28.7	25.8	27.25				
9 29	I	28.8	25.8	27.30	Mean		325 38 50	226 46 30
9 31	E	28.5	30.9	29.70				
9 32	E	28.5	30.9	29.70				

Mean scale readings.	d.
Erect	29.62
Inverted	27.22
Axis	28.42

	d.
Mean scale reading, erect	29.62
Axis	28.42
Scale—Axis	+1.20
Reduction to axis	+2′.40
Circle reading	226 46.5
Mag'c S. M. reading	226 48.9
Mark reading	325 38.8
Magnetic azimuth of mark	98 49.9
True azimuth of mark *	78 50.1
Magnetic declination, W	19 59.8
Diurnal variation	+5.5
Mean declination, W	20 05.3

Remarks:
Temp.: 27°.0 C.
Weather: Fair.
Torsion weight suspended 25 minutes.
1 division of scale=2′.0.

	h. m.
Mean chron. time	9 27.5
Chron. corr'n on L. M. T.	+27.3
Local mean time	9 55

* Counted from south around by west from 0° to 360°.

The azimuth of the mark and the chronometer correction on local mean time were obtained from the computation of the observations of the sun, reproduced on page 51. The magnetic south meridian reading subtracted from the mark reading gives the magnetic azimuth of the mark, and that subtracted from the true azimuth of the mark gives the magnetic declination, east when plus and west when minus. The correction for diurnal variation is supplied in the Office from the records of the nearest magnetic observatory, but its approximate value may be obtained (except for periods of magnetic storms) from

Table IX, which gives the average diurnal variation for different seasons of the year for the different observatories.

India Magnetic Survey pattern magnetometer.—Magnetometers of the type shown in Figure 4 are in use at three of the magnetic observatories of the Coast and Geodetic Survey. They are very well adapted

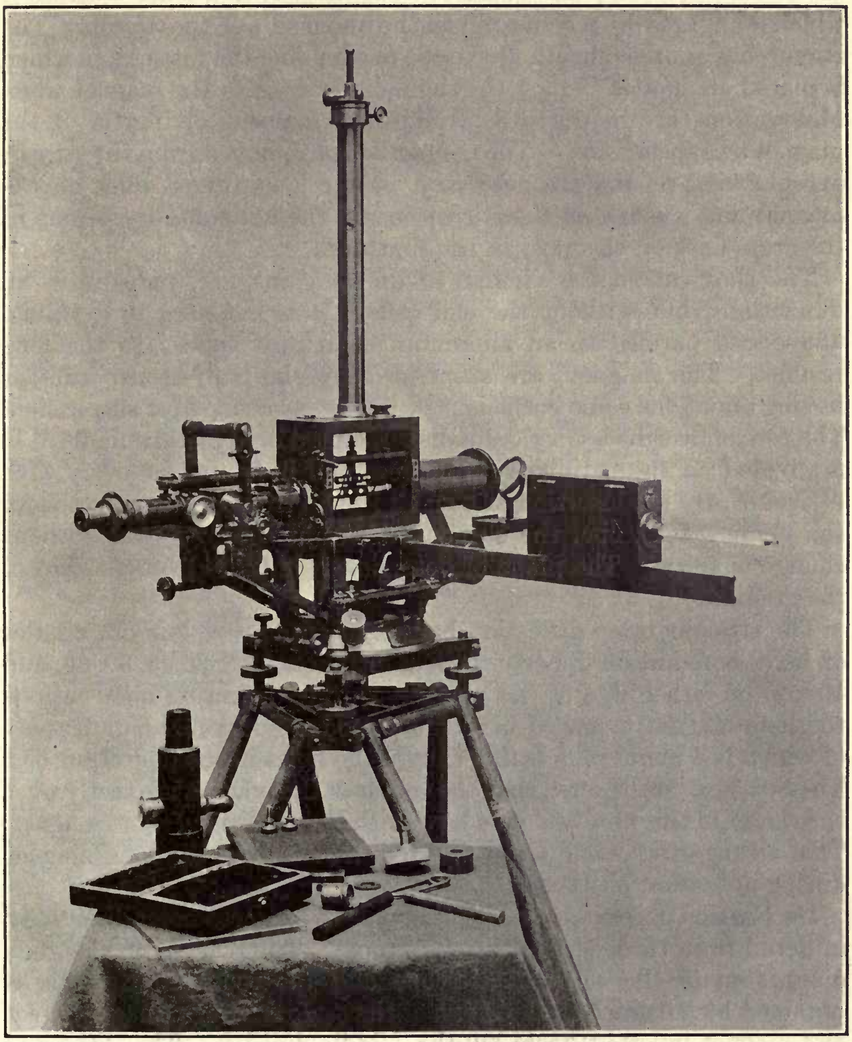

FIG. 4.—India Magnetic Survey pattern magnetometer.

for that purpose, but are rather too heavy for field work, although one of them has been so used for several years. This type of instrument was designed by Capt. H. A. Denholm Fraser, R. E., for use in the magnetic survey of India, and is a modification of the well-known Kew pattern. The long magnet is a hollow cylinder about 9 cm. long and 1 cm. in diameter, with an aluminum cell mounted externally at each end. The cell at the south end carries a piece of optical glass on which are engraved two lines at right angles to each other. The cell at the north end contains a collimating lens. On the glass diaphragm of the reading telescope there are two scales, one vertical and the other horizontal. Around the middle of the magnet is a shallow groove, which is engaged by a screw pin on the under side of the stirrup. The stirrup has another sheath above the one holding the magnet, in which is placed an inertia bar of the same dimensions as the magnet when observations are made to determine the moment of inertia of the magnet and suspension. The magnet is not removed from the stirrup except when repairs are necessary. Four longitudinal lines on the magnet and a mark on the stirrup insure the horizontality of one of the cross lines on the glass in the south cell.

The short magnet is similar to the long magnet, reduced in all dimensions, but without the end cells. It is mounted in a stirrup above and parallel to an aluminum collimator similar to the long magnet. The magnets are suspended by phosphor-bronze ribbons having about the same coefficient of torsion as a silk-fiber suspension. The torsion weight is a xylonite disk mounted on a metal spindle. It is divided on the periphery to degrees and is figured at every fifth division. By the insertion of a small lens in front of the objective of the reading telescope, the suspended weight may be read without change of focus. The plane of detorsion is indicated by the zero of the graduation.

The straight brass deflection bar is not graduated, but has a series of holes bored in its upper surface at distances 22.5, 26.25, 30, 35, and 40 cm. on either side of the center. During deflection observations the long magnet is placed in a small wooden box, on the under side of which is a metal plug fitting snugly the holes in the deflection bar. The box is so constructed that the center of the magnet is exactly over the center of this plug and on a level with the suspended short magnet. This arrangement eliminates all direct handling of the long magnet during deflection observations.

Declination observations with this type of magnetometer differ only in detail from those given in the example. The scale is on the glass diaphragm of the reading telescope and the reduction to axis is obtained by subtracting the mean of the scale readings (magnet erect and magnet inverted) from 50, the middle division. The difference between the erect and inverted readings is twice the angle between the geometric and magnetic axes of the magnet and should remain very nearly constant.

The angular value of one division of the scale may be obtained by pointings on a distant object instead of on the magnet.

Magnetometers of other design are so seldom used in the field work of the Survey that it is unnecessary to describe them in detail. Mention may be made of No. 21, a very small instrument weighing only 4 kg., similar to the one used in the magnetic survey of France and described by Mascart on page 212 of his "Traité de Magnétisme Terrestre," and No. 25, a combination instrument of the Prussian field magnetometer type, consisting of theodolite, magnetometer, declinometer, and dip circle, all arranged for mounting on the same base. A description of this instrument will be found in "Results of Observations made at the Coast and Geodetic Survey Magnetic Observatory at Sitka, Alaska, 1902–1904."

DECLINATION FROM HORIZONTAL INTENSITY OBSERVATIONS.

In the directions for determining horizontal intensity, given later on, it will be seen that provision is made for reading the scale of the magnet and the horizontal circle in connection with the observations of oscillations. This furnishes a check on the regular declination observations which immediately precede or follow, since the change in scale reading should correspond with the change in circle reading; or a value of the declination may be computed by assuming that the mark reading and the scale reading of the axis are the same as during the regular declination set. In the example given it was not necessary to shift the setting of the horizontal circle between declination and oscillation observations, since the axis reading was very near the middle division (30).

A value of declination may also be obtained from the two sets of deflections, provided the short magnet is erect in one set and inverted in the other, and provided also that the position of the instrument is not disturbed between a set of deflections and one of the declination sets, so that the mark reading may be assumed to be unchanged. The observations are so arranged that the horizontal circle is read when the magnet is deflected by approximately equal amounts in opposite directions from the magnetic meridian, and the mean of the readings therefore represents the reading of the magnetic south meridian, which combined with the mark reading gives the magnetic azimuth of the mark. In the sample set of deflections on page 78:

	°	′	″
Mean of 1, 4, 5, 8	226	29	36
2, 3, 6, 7	226	29	15
Magnetic south meridian reading	226	29	26
Mark reading, from declination above	325	38	50
Magnetic azimuth of mark	99	09	24

From the second set which followed with magnet inverted:

	° ′ ″
Mean of 1, 4, 5, 8	295 32 26
2, 3, 6, 7	295 32 25
Magnetic south meridian reading	295 32 26
Mark reading, from second declination set	34 08 20
Magnetic azimuth of mark	98 35 54
Mean of two sets	98 52 39
True azimuth of mark	78 50 05
Magnetic declination, W	20 02 34
Diurnal variation	+ 2 00
Mean declination, W	20 04 34

In this case the horizontal circle was shifted by means of the lower clamp between the two sets of deflections. The difference between the two values of the magnetic azimuth of the mark represents approximately twice the angular distance between the magnetic axis of the short magnet and the middle division of its scale.

B. WITH A COMPASS DECLINOMETER.

Two types of compass declinometer are in use in the Coast and Geodetic Survey. The older form is essentially an improved prismatic compass with slit and thread arrangement for pointing on the object selected as a mark. It consists of a cylindrical brass bowl resting on three foot screws, in the center of which is the pivot on which the needle rests. The needle is a flat (in the vertical plane) rectangular bar 9 cm. long, terminating in steel points. It is expanded at the center to inclose an agate cup which may be inserted from either side, thus making it possible to invert the needle. The change in balance of the needle with change in vertical intensity is corrected by means of a small weight which may be slid along the needle. The portion of the instrument which carries the alidade forms the cover of the bowl and may be taken off and reversed. The horizontal circle has a limb 12 cm. in diameter divided to 10′ and read by estimation to whole minutes. The instrument is provided with a circular level.

The compass declinometer is intended especially for use by triangulation parties, where the azimuth is known and time is not available for more extended magnetic observations. As the slit and thread arrangement can not be used for sighting on a very distant object, it is usual either to place a temporary reference mark on line from the triangulation station to a distant object by means of a theodolite or else to set up the compass declinometer accurately in line and use the triangulation station itself as the reference mark.

In a perfect instrument the point of support of the needle should be in the vertical of the center of graduation, and the three objects—the slit at the prism, the point of support, and the sight wire—should be in the same vertical plane. When this is not the case, the instrument will have an index correction, constant so long as the adjustment of the instrument remains unchanged, which must be determined at the beginning and end of a season's work at some place where the declination is known. Changing the position of the prism, i. e., moving it up or down on its support, if the latter be not truly vertical, will change the index correction. When making the observations to determine the index correction, therefore, the observer should mark the position of the prism and in subsequent observations should be sure that it is in the same position.

Form 38.

MAGNETIC DECLINATION.

Station, Cheltenham, Md. Date, Monday, February 28, 1910.
Compass Declinometer No. 741. Observer, J. E. Burbank.
Mark, Hill's barn cupola.

Chron. time, a. m.	Mark.	Circle direct. Needle direct.		Circle reversed. Needle inverted.		Mark.
		North end.	South end.	South end.	North end.	
h. m.	° ′	° ′	° ′	° ′	° ′	° ′
11 24	257 08	183 05	2 54	182 54	2 57	257 09
	77 06	3 04	182 54	2 53	182 57	77 06
	257 10	182 54	2 57	183 02	2 59	257 06
11 40	77 08	2 53	182 56	3 00	182 57	77 04
Means	257 08.0	182 59.0	182 55.2	182 57.2	182 57.5	257 06.2
p. m.						
h. m.						
16 13	17 06	302 50	122 49	122 40	302 38	17 03
	197 04	122 50	302 48	302 38	122 37	197 02
	17 04	302 48	122 49	122 48	302 35	17 04
16 28	197 02	122 49	302 48	302 44	122 34	197 02
Means	17 04.0	302 49.2	302 48.5	302 42.5	302 36.0	17 02.8

	h. m.
Chron. correction on standard 75th mer. time*	−2 25
Difference of longitude, 1° 50′	− 7
Chron. correction on local mean time*	−2 32

Local mean time.	*h. m.* 9 00	*h. m.* 13 48	Remarks.
	° ′	° ′	
Mark reading	257 07.1	17 03.4	
Needle reading	182 57.2	302 44.1	
Magnetic azimuth of mark	74 09.9	74 19.3	
True azimuth of mark†	68 51.3	68 51.3	
Magnetic declination, W	5 18.6	5 28.0	
Index correction	+14.4	+14.4	
Diurnal var. correction	+ 2.0	− 5.0	
Resulting declination, W	5 35.0	5 37.4	

* Plus when slow, minus when fast. † Counted from south around by west.

After leveling the instrument and adjusting the position of the prism, the verticality of the sight wire should be tested by means of a plumb line or the vertical edge of a building. The mark selected should be nearly in the horizon, so that the error due to lack of verticality of the wire may be as small as possible.

Place the needle on the lifter, close the box, and lower the needle gently onto the pivot. Adjust the balance if necessary by means of the sliding weight. The order of observations will then be as follows, both indices being read in each case: (1) Two pointings on the mark; (2) one pointing on the north end of the needle; (3) one pointing on the south end of the needle; (4) one pointing on the south end of the needle; (5) one pointing on the north end of the needle; (6) invert the needle and reverse the upper part of the instrument; (7) one pointing on the south end of the needle; (8) one pointing on the north end of the needle; (9) one pointing on the north end of the needle; (10) one pointing on the south end of the needle; (11) two pointings on the mark. The needle should be disturbed slightly between readings (3) and (4) and (8) and (9) by means of a piece of steel (screw driver or knife).

Record the times of beginning and ending of the pointings on the needle and give the correction of the timepiece on standard time. It is preferable to make observations both morning and afternoon at about the times when the easterly and westerly extremes of declination ordinarily occur or late in the afternoon when the reduction to mean of day is usually small. (See Table IX.) If time permits, three sets of observations should be made, shifting the position of the foot screws on the tripod between sets.

While in theory the above type of compass declinometer has many points to commend it, in practice it has not proved entirely satisfactory. In two cases the index correction has been found to be different for different positions of the needle with respect to the bowl, indicating the presence of magnetic impurities in the metal of which the bowls are constructed, although special tests have failed to detect the seat of the trouble.

In supplying the demand for additional compass declinometers, the satisfactory results which have been obtained in determining the declination with compass attachments fitted to dip circles have led to the construction of a new type of instrument from designs prepared by Mr. E. G. Fischer, Chief of the Instrument Division. It is nothing more than a compass needle with peep sights mounted on a graduated horizontal circle, but some of the details are novel and all have been worked out with great care. The base rests on three leveling screws, has double centers, and the horizontal circle is read by two verniers. This base supports a rectangular box, in which is mounted a compass needle about 6 inches long. At each end of the needle is a graduated arc, about 20° in extent, with the zero in the middle. Vertical peep sights are attached to the ends of the box, so that the zeros of the graduations and the point of support of the needle are in the vertical plane through the peep sights. The lifter of the needle is of special design, so arranged that the instrument can not be packed for shipment without first lifting the needle off the pivot. Observations with this type of instrument differ but little from those described in detail for the older form of declinometer.

The instrument should be set low enough to permit the observer to look directly down upon the needle when making the settings. After the instrument has been leveled and the sliding weight adjusted in position if necessary, the order of observations is as follows: (1) Two pointings on the mark, one direct and one reversed; (2) one reading, north end of needle set at zero; one reading, south end set at zero; one reading, south end set at zero; one reading, north end set at zero. In a similar manner (3) four readings with north end set 5° east of zero or south end 5° west of zero; (4) four readings with north end 5° west of zero or south end 5° east of zero; (5) four readings with the ends set at zero; (6) two pointings on the mark. Record must be made of the time of beginning and ending. It will be sufficient to read one vernier for settings on one end of the needle and the other vernier for settings on the other end. The needle should be lifted when pointing on the mark. The eye should be moved up and down the slit to insure accuracy of pointing.

Observations with the compass attachment of a dip circle (shown in Fig. 5) are made in the same manner and the method of computation is in each case the same as for the older form of declinometer. A sample set of observations is given below.

Form 38a.

MAGNETIC DECLINATION.

Station, Sweetwater, Tex.
Compass of dip circle No. 30.
Mark, Cupola of schoolhouse.

Date, January 17, 1910.
Observer, W. H. Burger.

Chron. time.	Mark, circle direct.	Needle set at 0°. set at 0°.		Needle set 5° right. set 5° left.		Mark, circle reversed.
		North end.	South end.	North end.	South end.	
h. m.	° ′	° ′	° ′	° ′	° ′	° ′
10 32	13 05	21 24	21 25	16 27	16 25	13 01
		23	23	26	24	
		23	26	26 19	26 18	
10 40	13 05	24	25	20	18	13 04
Means.	13 05.0	21 23.5	21 24.8	21 23.0	21 21.2	13 02.5

DETERMINATION OF THE DIP.

A. WITH A DIP CIRCLE.

The dip or inclination is usually measured by means of a *dip circle,* in which a magnetized needle is mounted in such a way as to swing in a vertical plane about an axle through its center of gravity. The form of dip circle in general use is the Kew pattern shown in Figure 5. The pivots of the needle rest on agate knife edges, the supports of which are horizontal or vertical according as the instrument is intended for use in high or low magnetic latitudes. The needle is placed in position by means of a lifter so arranged that when the needle is lowered onto the agate knife edges, the prolongation of the axis of the pivots passes through the center of the graduated vertical circle. The vertical circle is read by two verniers, and in older instruments is usually graduated from zero at either side to 90° at the top and bottom. Some of the more modern dip circles are graduated continuously from zero to 360°. To the frame carrying the verniers are attached two microscopes for pointing on the ends of the needle, so placed that when the circle reading is zero the line joining the microscopes is horizontal. On the frame carrying the microscopes are blocks for holding in position the needle used as a deflector in the determination of total intensity by Lloyd's method, so arranged that when the needle is in position its axis is at right angles to the line joining the microscopes. Four needles are usually provided, two for regular dip observations and two for the determination of total intensity.

Some of the newer dip circles of this pattern are provided with a compass needle mounted in a rectangular box, which may be placed on top of the dip circle as shown in Figure 5. The angle between the magnetic meridian as defined by the compass needle and the line to some mark of which the true bearing is known may be measured with the aid of peep sights.

In dip circles of the Lloyd-Creak pattern (Fig. 9), designed for observations on shipboard, but suitable also for land observations, the pivots of the needle rest in agate cups instead of on agate knife edges and the ends of the needle are in close proximity to the graduated circle so that the end of the needle and the adjacent graduation are seen through the reading microscope at the same time.

In dip circles of the Brunner pattern a movable graduated circle is immediately behind the needle and carries at the opposite extremities of a diameter two small concave mirrors, the centers of which are as far apart as the points of the needle. A setting is made by revolving the graduated circle until the point of the needle and its reflected image coincide. The angle of dip is then read off on a fixed vernier.

The adjustment of a dip circle is usually made with care in the instrument shop before the instrument is sent into the field and seldom requires attention in the course of a season's work. As cases may arise, however, where it is important to make adjustments in the field, the following directions are given.

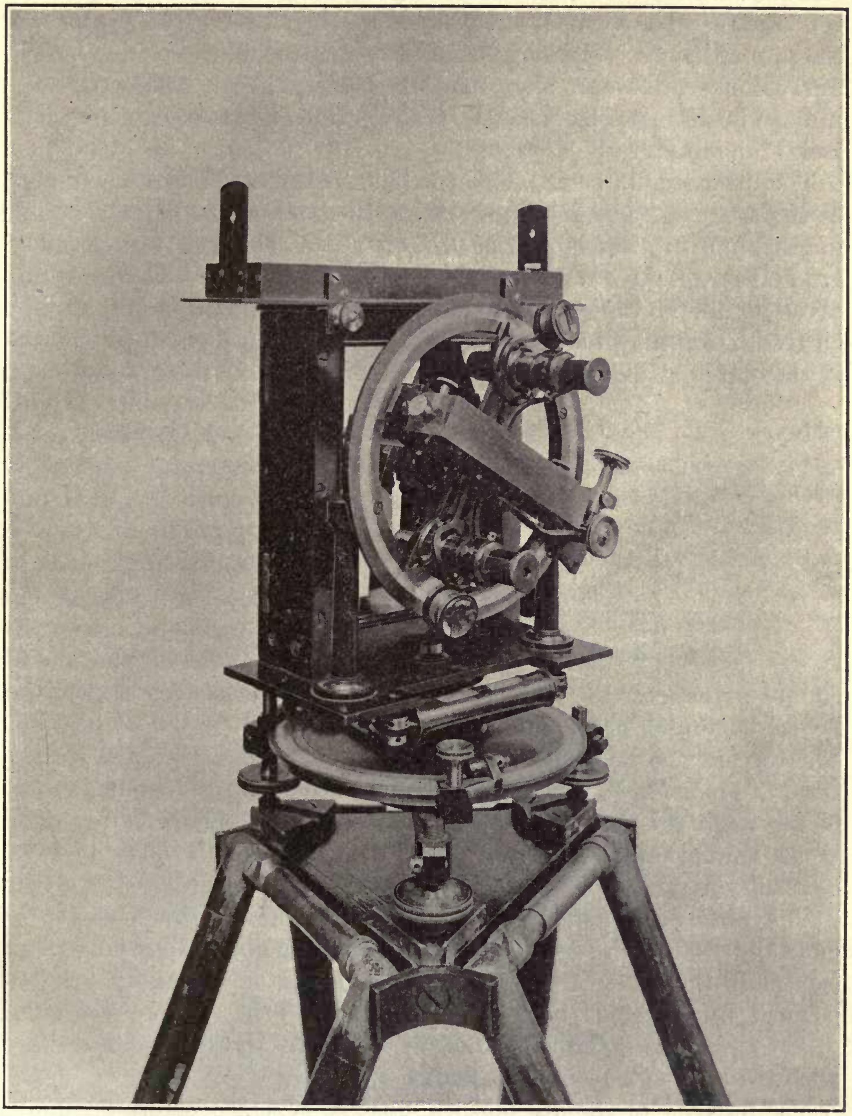

FIG. 5.—Kew pattern dip circle.

The bearing surfaces of the agate knife edges should lie in a horizontal plane which if produced would pass below the center of graduation of the vertical circle at a distance equal to the radius of the pivots of the needles. A small level is provided to assist in making this adjustment. The height of the agate surfaces with reference to the center of graduation may be tested by placing the needle in position and keeping it nearly horizontal by a strip of wood or piece of stiff paper under the north end. If the two ends read the same (or 180° apart, if the vertical circle is graduated to 360°) the needle is at the proper height. Readings should be made in both positions of the microscopes to be sure that they are placed exactly 180° apart and in both positions of the needle, face east and face west, to correct for lack of symmetry.

The lifter should be adjusted so that when the needle is lowered on to the agate surfaces its pivots will both touch at the same time and its axis of rotation if produced would pass through the center of graduation of the vertical circle, so that the needle will rotate in a plane parallel to the graduation. The vertical line through the center of graduation may be determined by suspending a small plumb bob at the end of a silk fiber, so that the fiber intersects the graduation at two points exactly 180° apart. A small movable hook is provided for this purpose in the top of most dip circles.

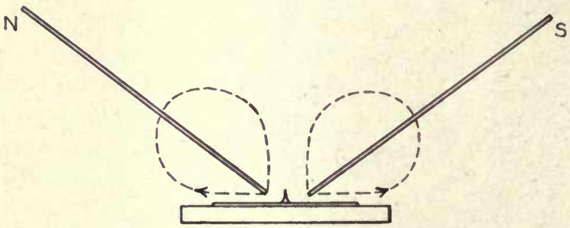

FIG. 6.—Remagnetization of dip needle.

The microscopes for pointing on the ends of the needle should be exactly 180° apart and should be focused for clear vision before beginning observations.

To avoid the necessity of carrying a separate tripod for the dip circle, an extra head is usually provided, which can be fastened on top of the magnetometer tripod when dip observations are to be made. When the dip circle has been placed in position its level is adjusted and the instrument leveled in the usual way.

The observer should have constantly in mind the necessity of guarding the needles from falls or other accidents and keeping them, especially the pivots, clean and free from rust. The pivots are best cleaned by sticking them into a piece of pith. Before beginning observations, the bearing surfaces of the agates should also be cleaned with the edge of a piece of paper or with pith.

The polarity of the dip needles must be reversed before beginning a set of observations as well as in the middle of the set. This operation is performed in the following manner: Place one needle on the reversing block after having determined which end is attracted downward (north end). Take one bar magnet north end down in one hand, and the other magnet south end down in the other hand, each inclined about 30° to the horizon. Draw the magnets lightly from center to end of the needle, the magnet with north end down resting on the end of the needle which was attracted downward. Make 5 strokes, then interchange the magnets and make 5 more. Then turn the needle over and repeat the operation, making 20 strokes in all. Care must be taken to stroke the same end of the needle with the north end of the magnets throughout the operation.

The next step is to determine the plane of the magnetic meridian and the corresponding reading of the horizontal circle. If the instrument is provided with a compass attachment, the magnetic meridian is readily determined by mounting the compass and turning the instrument until the compass needle points to zero. The instrument, that is, the plane of the vertical circle, is then in the magnetic meridian, and the reading of the horizontal circle, as well as the one differing by 180°, is the one at which the circle is to be set when making dip observations. Care must be taken to remove the compass attachment before the dip observations are made, otherwise the results will be vitiated.

In case no compass attachment is available, or in high magnetic latitudes, where the compass needle is sluggish, the magnetic meridian may be determined by taking advantage of the fact that when a dip needle is mounted in a plane at right angles to the magnetic meridian it will stand vertical. Raise the lifter, place one of the needles upon it with its "face" toward the reading microscopes. (The face of the needle is the side on which the letters A and B are engraved.) Set the upper vernier at 90° and place the instrument at right angles to the meridian, with the vertical circle toward the north. Lower the needle onto the agates and bring it nearly to rest by means of successive liftings and lowerings. Turn the instrument in azimuth until the swing of the upper end of the needle is bisected by the cross hair of the upper microscope, gently lifting and lowering the needle several times to make sure that it is swinging freely. Record the reading of the horizontal circle. Set the lower vernier at zero and repeat the operation, pointing on the lower end of the needle. Then turn the instrument 180° in azimuth and repeat the operations, beginning with the lower end of the needle. The mean of the four readings of the horizontal circle is the reading of the magnetic prime vertical, and as the circle is usually graduated by quadrants from 0° to 90°, the readings of the magnetic meridian will be the same. The dip observations proper may then be begun. It is usual to observe with two needles at each station, and the work is so arranged that the middle time of observation is the same for each needle. Observations should be begun with the needle which was magnetized first.

Place the instrument in the magnetic meridian, (vertical) circle east, needle face east, and reduce the swing of the needle to a small arc by means of successive liftings, noting at the same time whether the swing of the needle appears to be free and regular. (If such is not the case, the pivots and agates should be cleaned again.) Set on the upper (south) end of the needle and read the upper vernier; then set on the lower (north) end and read the lower vernier; then record the two readings. Better results are obtained if the needle is observed while swinging over a small arc, but it should not be disturbed between the readings of the two ends, so that the swing at the time of the first reading should be just sufficient to continue until the second has been made. The needle is then lifted and lowered and the two ends read in the reverse order. In general the two ends of the needle will not read the same, but the difference between the two should be nearly constant for a particular position of circle and needle. If such is not the case, or if the readings before and after lifting differ by as much as 8′, the readings should be repeated.

The circle is then turned 180° in azimuth and similar readings are taken in the position circle west, needle face west. Then the needle is turned over and observations made with circle west, needle face east, and finally the circle is reversed again and readings are taken in the fourth position circle east, needle face west. The same operations are then performed with needle No. 2.

Next the polarities of the two needles are reversed, so that the end which was down before will now be up, and a second half set of observations is made with No. 2, followed by a second half set with No. 1. The times of beginning and ending should be noted for each needle.

The mean of all the readings gives the resulting dip, unless there is much difference in the results before and after reversal of polarities, in which case a small correction is required (Table VIII), as explained on page 13. The computation is arranged for simplicity in such a way that means of two quantities are taken successively, so that the work may be performed mentally.

In case the vertical circle is graduated continuously from zero to 360°, it will be necessary to subtract the circle readings from 180° or 360° for circle west in order to get the angle of dip.

Form 42.

Magnetic Dip.

Station, Smyrna Mills, Me. Date, Friday, August 5, 1910.
Dip circle No. 5678. Needle No. 2. Observer, H. E. McComb.

End of needle marked A down.

Circle east.		Circle west.		Circle west.		Circle east.	
Needle face east.		Needle face west.		Needle face east.		Needle face west.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
75 19	75 20	75 39	75 41	75 45	75 39	74 52	75 04
22	22	41	43	48	41	51	03
75 20.5	75 21.0	75 40.0	75 42.0	75 46.5	75 40.0	74 51.5	75 03.5
75 20.8		75 41.0		75 43.2		74 57.5	
75 30.9				75 20.4			

Mean: 75° 25′.6

Polarities reversed. End of needle marked B down.

Circle east.		Circle west.		Circle west.		Circle east.	
Needle face east.		Needle face west.		Needle face east.		Needle face west.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
75 04	75 12	75 52	75 46	75 41	75 45	75 15	75 13
02	10	55	49	41	44	14	14
75 03.0	75 11.0	75 53.5	75 47.5	75 41.0	75 44.5	75 14.5	75 13.5
75 07.0		75 50.5		75 42.8		75 14.0	
75 28.8				75 28.4			

Mean: 75° 28′.6

Resulting dip: 75° 27′.1

	h. m.
Chron. time of beginning	2 10
" " " ending	2 32
Mean chronometer time	2 21
Chron. correction on L. M. T.	+ 27
Local mean time	2 48
	° ′
Magnetic meridian reads	55 47

Instrument in mag. prime vertical.

Vertical circle.	Needle.	Hor. circle readings.
		° ′
North	S. end at 90°	55 48
"	N. end at 90°	56 15
South	N. end at 90°	55 14
"	S. end at 90°	55 50
	Mean	55 47

B. WITH AN EARTH INDUCTOR.

A small earth inductor of the type designed by Wild is shown in Figure 7. The coil is rotated by means of a gear and piece of flexible shafting. A sudden starting or stopping of the rotation must be avoided, as it is apt to break the flexible shaft. The galvanometer should be mounted in a position where it can be observed conveniently while rotating the coil. The axis of the vertical circle is leveled by means of a stride level and the axis of the coil is placed in the magnetic meridian by means of a compass attachment. The observations for determining the dip are made in the following manner:

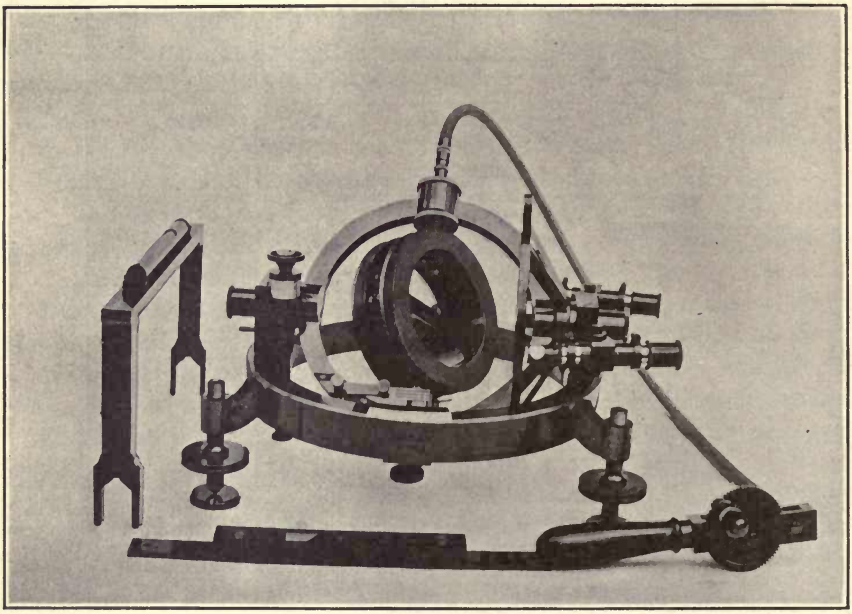

FIG. 7.—Wild pattern earth inductor.

(1) With vertical circle east, place the axis of the coil vertical by means of the level inside the coil and read the vertical circle, first with face of coil marked A east, then with face marked B east.

(2) Place the axis of coil approximately in the line of dip, rotate the coil, and observe the galvanometer. If the instrument is set accurately in the magnetic meridian, the galvanometer will be deflected steadily in one direction. If it is not quite in the meridian, the galvanometer will be deflected a small amount in one direction, and then as the speed of rotation increases will go off in the opposite direction. By successive trials find the setting at which no deflection of the galvanometer is produced when the coil is rotated. Record the time and the reading of the vertical circle. Rotate the coil in the opposite direction, make another setting, and read the vertical circle. Make

two more settings, one for rotation in each direction. The circle should be clamped when the coil is being rotated. After the first reading the changes in setting can be made with the tangent screw. The operation will be facilitated if the crank for rotating the coil is supported so that one hand may be free to move the tangent screw while the other rotates the coil.

(3) Place the axis of the coil vertical again, and read the vertical circle in two positions of the coil.

(4), (5), and (6) Proceed in the same manner with vertical circle west.

The difference between the circle readings for axis vertical and axis inclined gives the co-dip. The form of observation and computation is shown in the following example:

Form 407.

MAGNETIC DIP.

Station, Sitka, Alaska. Date, February 5, 1908.
Earth inductor No. 2. Observer, H. M. W. Edmonds.

Magnetic meridian reads: 10° 40′.

Vertical circle east.					Vertical circle west.				
AXIS VERTICAL.									
Coil.	(Order).	A.	B.	Mean.	Coil.	(Order).	A.	B.	Mean.
		° ′	′	′			° ′	′	′
A–E	Begin	0 11.0	10.9	11.0	A–E	Begin	0 09.2	09.2	09.2
B–E	(1)	10.8	10.8	10.8	B–E	(4)	09.9	09.9	09.9
A–E	End	11.0	10.9	11.0	A–E	End	09.9	09.8	09.8
B–E	(3)	10.9	10.8	10.8	B–E	(6)	09.9	09.7	09.8
			Mean	0 10.9				Mean	0 09.7
AXIS INCLINED.									
Rot.	Chron.	A.	B.	Mean.	Rot.	Chron.	A.	B.	Mean.
	h. m.	° ′	′	′		h. m.	° ′	′	′
+	9 34	344 44.5	44.1	44.3	+	9 46	15 32.1	32.1	32.1
−	(2)	52.0	52.0	52.0	−	(5)	33.0	33.0	33.0
+		45.7	45.3	45.5	+		33.0	33.0	33.0
−	9 40	51.8	51.8	51.8	−	9 49	31.9	32.0	32.0
Mean	9 37			344 48.4	Mean	9 48			15 32.5
			Dip	74 37.5				Dip	74 37.2
Mean dip 74° 37′.35									

	h. m.
Mean chronometer time..	9 42
Chronometer correction..	− 5
Local mean time.........	9 37

DETERMINATION OF THE HORIZONTAL INTENSITY.

As already explained, the determination of the horizontal intensity involves two operations called "oscillations" and "deflections." The observations at a station usually comprise two sets of each, arranged in the order: Oscillations, deflections, deflections, oscillations. They are made with a magnetometer, two types of which have been described, and as they usually follow a set of declination observations it may be assumed that the instrument is in adjustment, that the torsion has been removed from the fibers, and that the long magnet is suspended.

TORSION OBSERVATIONS.

Point approximately on the middle division of the scale of the magnet, reduce the arc of vibration to two divisions or less, and read the horizontal circle. Read the torsion circle at the top of the suspension tube and the scale of the magnet at the extremes of its swing, as in declination observations, and record the readings in the place provided on the form. Turn the torsion head 90° to the right and read the scale of the magnet. Turn the torsion head 180° to the left (i. e., 90° to the left of its original position) and again read the scale. Turn the torsion head 90° to the right and read the scale. The torsion circle now reads the same as at the beginning, and the last scale reading should be very nearly the same as the first. The differences between successive scale readings give the effect of 90°, 180°, and 90° of torsion, respectively, in scale divisions, and their sum divided by four and multiplied by the arc value of one division of the magnet scale is the average effect of 90° of torsion, the quantity h required to correct the time of one oscillation for effect of torsion.

OSCILLATIONS.

The oscillations are usually arranged in such a way as to give six or eight independent determinations of the time of a selected number of oscillations, which, for convenience in computing, should be some multiple of 10. Increase the arc of vibration to about 20 divisions, 10 on either side of the middle, and determine the approximate time of one oscillation by counting the number of seconds required for four or six oscillations, and from that compute the time, approximating half a minute, which would be required for some odd number of oscillations. In the example six oscillations took about 34 seconds. Hence the time of five oscillations would be about 28 seconds. The observer then arranged his program to observe every fifth oscillation from 0 to 35 and from 50 to 85, thus obtaining eight independent determinations of the time of 50 oscillations. When the observing program has been outlined in the first column of the form the succeeding operations are as follows: Read the thermometer and the scale of the magnet. Note and record on the first line of the second column of the form the time when the middle division of the scale of the magnet crosses the vertical line of the reading telescope, the magnet swinging from left to right. About 28 seconds later note and record the time when the middle division of the scale crosses the vertical line, magnet swinging from right to left, and so on at intervals of about 28 seconds until eight readings have been taken. Then read the thermometer and scale again. Compute approximately the time when the fiftieth oscillation may be expected, and when that time arrives begin a second series of eight readings at intervals of about 28 seconds. At the close read the thermometer and the scale of the magnet again. This completes a set of oscillations. By this method it is necessary to look in the reading telescope for only a few seconds at the time of each observation. A few seconds before the predicted time of transit the observer picks up the beat of the chronometer and begins to count half seconds and then looks into the reading telescope and waits for the transit to occur. Thus, for the fifth oscillation he might pick up the beat at $9^h\ 38^m\ 35^s$ and count: Half-six—half-seven—half-eight—half-nine—half-ten—half-one—half-two—half-three, the transit occurring between *half* and *three.* The fraction of the half second can best be estimated by noting mentally the relative position of the middle division and the vertical line of the telescope for the beats just before and after the transit and dividing up the half second in the same proportion that the space is divided by the position at transit. It will usually be possible to hear the beat of the chronometer while observing, but in case this is prevented by noise the observer can with a little practice learn to count the half seconds accurately without hearing the tick for the short interval involved. The chronometer should be kept far enough from the magnet to guard against the disturbing effect of the steel spring, etc.

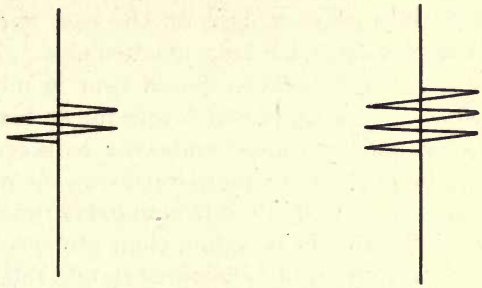

FIG. 8.—Four and six oscillations.

DEFLECTIONS.

Place the deflection bars in position, remove the long magnet from the stirrup and suspend the short magnet in its place with scale erect, taking care to keep the two at least 20 cm. apart. Remove the thermometer from the magnet house and plug up the hole. Remove the thermometer from its case (if it is in one) and place it inside the east deflection bar. Place the long magnet with scale erect and north end east at the shorter distance on the east bar and the torsion weight as a counterpoise on the west bar. Be sure that the short magnet is in the same horizontal plane with long magnet. Point on the middle division of the scale of the suspended magnet, checking its swing to about two divisions of the scale, and read the horizontal circle. Move the long magnet out to the longer distance and again point on the middle division of the scale of the suspended magnet and read the horizontal circle. Turn the long magnet end for end and repeat the pointing and reading; then move it up to the shorter distance and make a fourth pointing and reading. Remove the thermometer from the east bar, read it, and place it inside the west bar. Place the long magnet with north end west at the shorter distance on the west bar and the torsion weight on the east bar. The subsequent procedure is the same as for long magnet east. Read the thermometer at the close. The observer should bear in mind that it is the temperature of the long magnet which is required both in oscillations and in deflections, and he should endeavor to place the thermometer so that it will be of the same temperature as the magnet. If the temperature is changing rapidly or if it is materially different on the two bars, more readings should be taken than are specified above.

A second set of deflections should follow immediately after the first, but with both magnets inverted and reversing the order of the positions of the long magnet. At its close, return the short magnet, deflection bars, and torsion weight to the magnetometer case, suspend the long magnet *inverted*, return the thermometer to its case and to the hole in the magnet house, and make a second set of oscillations.

A second set of declination observations usually follows, but the horizontal circle should first be shifted so as to bring the readings on a different part of the graduation.

Form 41.

HORIZONTAL INTENSITY. OSCILLATIONS.

Station, Smyrna Mills, Me. Date, Friday, August 5, 1910.
Magnetometer No. 20. Magnet 20 L. Observer, H. E. McComb.
Chronometer, 245, daily rate losing $0^s.2$ on mean time.

Number of oscillations.	Chronometer time.	Temp. t′	Extreme scale readings.		Circle reading.
	h. m. s.	°	*d.*	*d.*	° ′ ″
0	9 38 14.4	27.7	12.0	48.0	226 45 20
5	38 42.9				46 45 40
10	39 11.6				
15	39 40.1				226 45 30
20	40 08.6				
25	40 37.2				Time of 50 oscillations.
30	41 05.7				
35	41 34.3				
		28.1	14.0	45.5	
					m. s.
50	9 42 59.9				4 45.5
55	43 28.5				45.6
60	43 56.9				45.3
65	44 25.7				45.6
70	44 54.0				45.4
75	45 22.7				45.5
80	45 51.0				45.3
85	46 19.7	28.7	15.7	43.5	45.4
	Means.	28.17	13.90	45.67	4 45.45

Formula: $MH=\pi^2 K \div \left[T^2 \left(\frac{5400}{5400-h}\right) \left(1+(t-t')\,q\right) \left(1+\mu\frac{H}{M}\right) \right]$

Torsion observations.

Torsion circle.	Scale.		Mean.	Diff's.
°	*d.*	*d.*	*d.*	*d.*
80	28.8	30.0	29.40	
170	27.2	30.0	28.60	0.80
350	29.5	30.7	30.10	1.50
80	28.9	30.0	29.45	0.65

Mean $h=0^d.74=1'.48$

$(t-t')=-0.°87$

One division of scale=2′.00.

Time of 1 oscil.	5.70900
Corr'n for rate *	+ 1
T	5.70901
Log T^2	1.51312
" $\left(\frac{5400}{5400-h}\right)$	12
" $[1+(t-t')q]$	− 18
" $\left(1+\mu\frac{H}{M}\right)$	123
" Divisor	1.51429
" $\pi^2 K$	3.24733
" MH	1.73304

* Plus for losing rate and minus for gaining rate.

Form 39.

HORIZONTAL INTENSITY. DEFLECTIONS.

Station, Smyrna Mills, Me.
Magnetometer, No. 20.
Long magnet deflecting.

Date, Friday, August 5, 1910.
Observer, H. E. McComb.
Short magnet suspended.

Magnet.	North end.	Circle readings. I. Distance r=30 cm. No.	A.	B.	Mean.	II. Distance r=40 cm. No.	A.	B.	Mean.
			° ′ ″	′ ″	′ ″		° ′ ″	′ ″	′ ″
East.	E.	1	236 57 10	57 20	57 15	2	230 52 20	52 30	52 25
	W.	4	216 04 40	04 50	04 45	3	222 07 50	08 00	07 55
	2 u		20 52 30				8 44 30		
West.	W.	5	216 17 40	17 50	17 45	6	222 11 10	11 20	11 15
	E.	8	236 38 30	38 50	38 40	7	230 45 20	45 30	45 25
	2 u		20 20 55				8 34 10		

Formulas: $\frac{H}{M}=\left[\frac{2}{r^3}\left(1+\frac{P}{r^2}+\frac{Q}{r^4}\right)\left(1-\frac{2\mu}{r^3}\right)\right]\frac{1}{\operatorname{Sin} u}=\frac{C}{\operatorname{Sin} u}$

$\log H=\frac{1}{2}\left(\log\frac{H}{M}+\log MH\right)$

	I.	II.	Set.	I.	II.
	° ′ ″	° ′ ″			
2 u (mean)	20 36 42	8 39 20	log C	$\bar{5}$.86935	$\bar{5}$.49414
u	10 18 21	4 19 40	" Sin u	$\bar{9}$.25262	$\bar{8}$.87773
			" $\frac{H}{M}$	$\bar{6}$.61673	$\bar{6}$.61641
Value of log MH from oscillations:			" MH	1.73304	1.73304
			" H	$\bar{9}$.17488	$\bar{9}$.17472
Began at	$h.$ $m.$ 9 55	Temp. 26.6°	H	.14958	.14953
Ended at	10 13	" 28.0	log M	2.55816	2.55832
			Red'n to 20°	+152	+152
Mean	10 04	t= 27.3			
Chron. corr'n	+27		log M_{20}	2.55968	2.55984
L. M. T.	10 31		Mean	2.55976	

COMPUTATION.

The computation involves simply the substitution of the observed quantities and the instrumental constants in the formulas and requires little explanation. The observer is supplied with a table of constants which gives, for the magnetometer he is to use, the results of the special observations made for determining the scale value, moment of inertia, temperature coefficient, distribution coefficients, and induction coefficient of the long magnet and the deflection distances, and the combination of the last four (log C) which enters into the deflection formula. For the magnetometer used in the example, this table was as follows:

Constants of magnetometer No. 20.

One division of scale of long magnet = 2′.00.

Deflection distances.		log C at 20° C.
cm. 29.9874	log = 1.47694	$\bar{5}.86953$
40.0054	1.60212	$\bar{5}.49432$

For an increase of 1° C. in temperature log C must be diminished by 0.000025. (See Table VII.)

Temperature coefficient $q = 0.00048$ for 1° C.; $\log(1+q) = 0.000208$

Distribution coefficient $P = -0.955$

Induction factor $\mu = 6.81$ $\log \mu = 0.833$

When $\log \frac{H}{M} =$	$\log\left(1+\mu\frac{H}{M}\right) =$
$\bar{6}.50$	0.00093
$\bar{6}.55$	105
$\bar{6}.60$	118
$\bar{6}.65$	132
$\bar{6}.70$	148
$\bar{6}.75$	166
$\bar{6}.80$	186

Moment of inertia K	Temp.	$\log \pi^2 K$
	0° C.	3. 24703
	10	713
	20	724
	30	735
	40	745

Computing the elapsed times between oscillations 0 and 50, 5 and 55, 10 and 60, etc., gives 8 independent values of the time of 50 oscillations. The mean of these 8 quantities divided by 50 is the time of 1 oscillation, T, which must, however, be corrected for the rate of the chronometer. The loss in 24 hours being $0^s.2$, the loss in $5^s.7$ would be $0.2 \times 5.7 \div 86400$, or, since the correction to the fifth decimal place is desired, $0.2 \times 5.7 \times 1.16$. Table V gives the value of this correction for different rates and different times of oscillation. The formula for MH is arranged for logarithmic computation, so that $\log MH = \log \pi^2 K - \left[\log T^2 + \log\left(\frac{5400}{5400-h}\right) + \log\left(1+(t-t')\,q\right) + \log\left(1+\mu\frac{H}{M}\right)\right]$. As the second and third factors of the divisor never differ much from unity, their logarithms are always nearly zero and it may be assumed without appreciable error that the logarithm varies directly as the variable part of the factor, i. e.:

$$\log [5400 \div (5400 - h)] = h \log [5400 \div (5400 - 1)]$$

$$\log [1 + (t - t') q] = (t - t') \log (1 + q)$$

Now $\log (5400 \div 5399) = 0.00008$. Hence the torsion correction, expressed in units of the fifth decimal place of the logarithm is found by multiplying by 8 the effect of 90° of torsion. In the example, $h = 1'.48$ and the corrective term is therefore .00012. The value of this factor may also be obtained from Table VI. In the example $t - t' = 27.°30 - 28.°17 = -0.°87$. From the table of constants for this instrument it will be seen that

$$\log (1 + q) = 0.000208$$

hence $$\log [1 + (t - t') q] = .000208\ (-0.87) = -.00018$$

A table may readily be prepared for a particular instrument giving the value of this factor for different values of $(t - t')$. The value of $\log \left(1 + \mu \frac{H}{M}\right)$ will be found in the table of constants for different values of $\log \frac{H}{M}$, and the value of $\log \frac{H}{M}$ is obtained from the computation of deflections. The value of $\log \pi^2 K$ for the temperature of the oscillations is found by interpolation from the table of constants. When the computation has been completed, the value of $\log M H$ is carried forward to the deflection form.

The differences between the pairs of circle readings at the two distances give two values of $2u$, double the deflection angle, for each distance, from which the values of u are obtained. The values of $\log C$ for the two deflection distances are given in the table of constants for the temperature 20° C. In the example the deflections were made at a temperature 27.°30 C. Hence the tabular values of $\log C$ must be decreased by $0.000025\ (27.30 - 20) = 0.00018$. The values of $\log \frac{H}{M}$ are then obtained by subtracting $\log \sin u$ from $\log C$. In good work the two values seldom differ by more than 0.00050. Should they differ by as much as 0.00100 the computation should be revised and if no mistake is found the observations should be repeated.

The computation of H and $\log M$ from $\log H M$ and $\log \frac{H}{M}$ follows. The resulting values of $\log M$ are for the temperature of deflections, 27.°30. The magnetic moment of a magnet varies with temperature, as we have seen, and in order to compare the values obtained at different times it is necessary to reduce all results to the same temperature. 20° centigrade has been adopted as a standard and all values of log M are reduced to that temperature. For practical purposes this may be done by means of the formula

$$\log M_{20} = \log M + (t - 20°) \log (1 + q)$$

In this case $(t - 20°) = 7.°3$ and $\log (1 + q) = 0.000208$. Hence the correction to be applied to log M is $+0.00152$.

Experience has shown that when a magnet is first magnetized its magnetic moment decreases quite rapidly. The rate of loss of magnetism gradually diminishes and after a few years becomes very small. A comparison of the values of log M_{20} obtained at different times in the course of a season's work is therefore valuable for several reasons. (1) It furnishes a test of the accuracy of the horizontal intensity determinations and sometimes leads to the detection of errors of observation or computation. (2) It furnishes the means of correcting the adopted value of temperature coefficient if there is considerable variation in temperature involved in the series of observations. (3) An accident to the magnet, such as a fall, or improper packing for transportation, will usually be revealed by a sudden decrease in log M_{20}.

DETERMINATION OF THE TOTAL INTENSITY.

The determination of total intensity with a dip circle by Lloyd's method involves two kinds of observations: Dip with loaded needle, and deflections. As the accuracy of the method depends upon the constancy of the condition of the needles between the time of standardization and the time of observation, every care must be taken to secure that constancy. The needles must never be remagnetized and must be kept from close proximity to disturbing influences. The weight used in the standardization observations should be left in place in the needle and any possibility of bending avoided. When the standardization observations are made, that weight should be selected which will be best suited to the region in which the instrument is to be used. The weight should be in the south end for places north of the magnetic equator and in the north end for places south of the magnetic equator, so that its effect will be to diminish the true dip. In the formula involved, $F = C\sqrt{\cos I' \csc u \csc u'}$, I' is the dip with loaded needle, u is the angle of deflection, and $u' = I - I'$. The effect on the result of an error in an observed value of I' will tend to diminish as I' approaches zero and u' approaches 90°. The best approximation to these limits is usually secured by using a weight sufficient to cause the loaded end of the needle to dip by a small amount, so that I' differs from zero by about the same amount that u' differs from 90°.

If the season's work covers such a large range of dip that the weight used during standardization can not be used throughout, or if for some other reason a change in the weight becomes necessary, the change should be made, if possible, at a place where observations can be made before the change as well as after.

The remarks regarding care and cleaning of needles and agates and adjustment of the dip circle made in connection with determination of the dip apply with equal force here. The instrument having been leveled and placed in the magnetic meridian, the observations of dip with loaded needle follow in each of the four positions: Circle east, needle face east; circle west, needle face west; circle west, needle face east; circle east, needle face west, in the same manner as for regular dip observations. If the south end is below the horizon the dip is regarded as negative. The loaded needle is then fastened in the place provided between the reading microscopes, "face" out, and covered by the brass shield. The other (lighter) intensity needle is placed on the lifter face east and lowered on to the agate supports. As the microscopes are turned in order to make a pointing on the suspended needle, carrying with them the deflecting needle, it will be found that there are two positions in which the suspended needle may be pointed upon by the microscopes, in one of which it is deflected toward the vertical and in the other away from the vertical. The microscope which in one case points on the north end of the needle will in the other case point on the south end. The microscopes are considered direct (D) when the south (upper) end of the suspended needle is deflected toward the right and reversed (R) when it is deflected toward the left. The angular difference between the two positions of the needle is $2u$, twice the angle of deflection. It may happen that the suspended needle will be deflected out of one quadrant into the adjoining one. In a dip circle where the vertical circle is graduated in quadrants from 0° in the horizon to 90° at the top and bottom, this fact must be noted in the record in order that the deflection angle may be computed correctly. Thus, for a dip of 70° and a deflection angle of 30° the circle readings would be 40° in the same quadrant and 80° in the next.

Deflection observations are made with microscopes D and R in each of the four positions: Circle east, needle face east; circle west, needle face west; circle west, needle face east; circle east, needle face west.

A second set of dip with loaded needle, similar to the first, is then made. In the intensity observations, as in regular dip, two pointings on each end of the needle are to be made in each position, the needle being lifted between.

In the Lloyd-Creak form of dip circle the needle is supported in agate cups, and before a reading is taken the needle must be jarred to a position of equilibrium by rubbing or tapping a metal point on top of the instrument with an ivory scraper.

A value of dip may be obtained from the deflection observations, since the suspended needle is deflected by approximately equal amounts in opposite directions from its undeflected position.

A sample set of observations and computation is given below. When the vertical circle is graduated from zero at the sides to 90° at the top and bottom and the needle lies in the same quadrant for both positions of the microscopes, direct and reversed, half the difference of the two circle readings gives the deflection angle and half their sum gives the dip. When the needle is in one quadrant for microscopes direct and the adjacent one for microscopes reversed,

$$u = 90° - \frac{D+R}{2} \quad \text{and} \quad I = 90° - \frac{D-R}{2}$$

When the vertical circle is graduated continuously from 0° to 360°, the readings with circle west are to be substracted from 180° in taking the means.

Then

$$u = \frac{D-R}{2} \quad \text{and} \quad I = \frac{D+R}{2}$$

For obtaining $u' = I - I'$ the best available value of I must be used. This is generally the mean of the results with the two regular dip needles, with the instrumental corrections applied. These corrections and the value of $\log C$ are determined at some place where the dip and horizontal intensity have been accurately determined by other means, and are usually supplied to the observer from the office. The formula arranged for computation by logarithms is:

$$\log F = \log C + \frac{\log \cos I' + \log \csc u + \log \csc u'}{2}$$

As it usually happens that the deflection angle is different for the two halves of the deflection set, the form is arranged for computing the two halves separately and two values of $\log C$ are determined to correspond. The form is also arranged to compute the horizontal intensity from the formula $H = F \cos I$.

Form 389.

Total Intensity. **Dip with Loaded Needle.**

Station, Fernandina, Fla. Date, April 14, 1910

Observer, S. S. Winslow.

Dip circle No. 35. Needle No. 4. Weight No. 6.

End of needle marked B north. North end* up.

Circle east.		Circle west.		Circle west.		Circle east.	
Needle face east.		Needle face west.		Needle face east.		Needle face west.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
153 35	333 38	26 24	206 27	25 40	205 32	154 08	334 02
38	42	25	30	40	35	08	05
36.5	40.0	24.5	28.5	40.0	33.5	08.0	03.5
−26 21.8		−26 26.5		−25 36.8		−25 54.2	
−26 24.2				−25 45.5			
Mean I', Set 1. −26° 04′.8				$u'=I-I'=88°$ 29′.3			

Circle east.		Circle west.		Circle west.		Circle east.	
Needle face east.		Needle face west.		Needle face east.		Needle face west.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
153 28	333 30	26 31	206 25	25 46	205 40	154 10	334 00
25	27	28	25	45	38	08	333 58
26.5	28.5	29.5	25.0	45.5	39.0	09.0	59.0
−26 32.5		−26 27.2		−25 42.2		−25 56.0	
−26 29.9				−25 49.1			
Mean I', Set 2. −26° 09′.5				$u'=I-I'=88°$ 34′.0			

	Chron. time.	Temp.	Remarks:
	h. m.	°	
Beginning	9 55	20.7	
Ending	10 21	20.8	
Mean	10 08	20.75	
Corr'n on L. M. T.	+23		
L. M. T.	10 31		
		° ′	
Magnetic meridian reads		84 17	

* Note whether north end is up or down. Do not reverse polarity.

Form 389.

TOTAL INTENSITY — **DEFLECTIONS.**

Station, Fernandina, Fla. Date, April 14, 1910.
Dip circle No. 35. Needle No. 4 deflecting, No. 3 suspended.

Circle east, needle face east.				Circle west, needle face west.			
D.*		R.*		R.*		D.*	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
274 17	94 21	211 30	31 33	266 58	87 00	329 22	149 23
15	25	28	30	55	86 58	23	23
16.0	23.0	29.0	31.5	56.5	59.0	22.5	23.0
94 19.5		31 30.2		93 02.2		30 37.2	
125 49.7		62 49.3		62 25.0		123 39.4	
62 54.9		31 24.6		31 12.5		61 49.7	
I = 62 22.3				*u* = 31 18.6			

Circle west, needle face east.				Circle east, needle face west.			
D.		R.		R.		D.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
329 02	149 10	265 24	85 36	210 30	30 38	273 10	93 29
03	10	27	32	28	40	15	30
02.5	10.0	25.5	34.0	29.0	39.0	12.5	29.5
30 53.8		94 30.2		30 34.0		93 21.0	
125 24.0		63 36.4		62 47.0		123 55.0	
62 42.0		31 48.2		31 23.5		61 57.5	
I = 62 19.8				*u* = 31 35.9			

	Chron. time.	Temp.
	h. *m.*	°
Beginning	10 05	21.0
Ending	10 20	20.8
Mean	10 12	20.9
Corr'n on L. M. T.	+23	
L. M. T.	10 35	

		° ′
I from deflections		62 21.0
I from regular dip needles	No. 1	62 22.3
	No. 2	62 26.7

Computation of *F* and *H*.		
log cos *I′*	9.95336	9.95307
" csc *u*	0.28427	0.28070
" csc *u′*	0.00015	0.00014
Sum	0.23778	0.23391
Half sum	0.11889	0.11696
log *C*	9.62333	9.62497
" *F*	9.74222	9.74193
Mean	9.74208	*F*=.55218
log cos *I*	9.66574	
" *H*	9.40782	*H*=.25575

* If the vertical circle is graduated in quadrants, note whether the upper (south) end of the suspended needle is north or south of the vertical.

DIRECTIONS FOR OBSERVATIONS AT SEA.

INTRODUCTION.

The instruments and methods employed for determining the magnetic elements on land require a number of modifications for observations on board ship. On account of the instability of the ship as an observing platform, a magnetometer with fiber suspension can not be used and in the dip circle agate cups take the place of agate knife edges. The instruments must be mounted in gimbals in order that they may remain approximately level in spite of the motion of the ship. It is usual to determine the magnetic declination by means of the standard compass and an azimuth circle, and the dip and total intensity by means of a Lloyd-Creak dip circle.

On account of the disturbing effect of the iron and steel which enter more and more into the construction of modern ships, the direct results of magnetic observations on shipboard are different for different headings of the ship, since they represent the combined effect of the earth's magnetism and the ship's magnetism, and means must be provided to separate the resultant into its component parts. It is customary, therefore, to make observations on 8 or 16 equidistant headings while steaming in a circle, first in one direction and then in the other. As a complete determination of dip and intensity on each heading of the forward and back swings would consume too much time, the practice has been adopted by the Coast and Geodetic Survey of observing deflections alone while swinging ship in one direction and loaded dip alone while swinging in the opposite direction. In addition to the total intensity derived from the combination of these observations, a value of dip on each heading results from the deflection observations.

The determination of declination, dip, and total intensity at sea requires, first, that observations be made with the dip circle at a base station on shore at the beginning and end of the cruise, to determine the intensity constant for the particular weight used at sea and the correction to the dip as derived from the deflection observations; and, second, that the ship be swung at the beginning and end of the cruise (and, if possible, in the highest and lowest latitudes reached) at a place near shore where the declination, dip, and intensity are known with reasonable accuracy from shore observations, in order to determine the deviations of the standard compass and of dip and intensity at the dip circle position.

For the general theory of the analysis of a ship's magnetism the reader is referred to the various publications of the hydrographic offices of different nations, at least one of which is to be found on almost every ship. (E. g. "Practical Problems and the Compensation of the Compass in the United States Navy;" (British) "Admiralty Manual for the Deviations of the Compass;" "Der Kompass an Bord," issued by the Deutsche Seewarte, etc.)

DECLINATION.

The amount by which the compass needle points east or west of *true* north is called the *compass error.*

The amount by which the compass needle points east or west of *magnetic* north is called the *deviation.* In each case east is considered positive and west negative. As the angle between the true meridian and the magnetic meridian is the magnetic declination, it follows that:

$$\text{Compass error} - \text{Declination} = \text{Deviation}.$$

Hence for the determination of the declination on board ship it is necessary to know the compass error and the deviation. The deviation may be represented approximately by an equation of the form

$$\text{Deviation} = A + B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

in which ζ is the magnetic heading of the ship, counted from north around by east. The second member of this equation may be divided into three parts: A, which is constant for all headings; ($B \sin \zeta + C \cos \zeta$), called the *semicircular* deviation, the values on two headings 180° apart being equal but of opposite sign; ($D \sin 2\zeta + E \cos 2\zeta$), called the *quadrantal* deviation, the values on two headings 90° apart being equal but of opposite sign. In theory, the determination of the deviations on any five headings will give five equations from which to compute the five coefficients A, B, C, D, E. In practice, however, it is found that satisfactory results can not be obtained unless observations are made on a greater number of headings properly distributed. It is apparent that when observations are made on 8 or 16 equidistant headings, the mean of the deviations will be A, the constant part of the deviation, and the computation of the other coefficients will be much simplified. For observations made in this way near shore where the declination is known;

$$\text{Mean compass error} - \text{Declination} = A$$

and for observations at sea when A has been determined:

$$\text{Mean compass error} - A = \text{Declination}.$$

From this it will be seen that when observations are made on a multiple of four equidistant headings it is not necessary to compute the coefficients B, C, D, E in order to determine the declination, but inasmuch as the deviations on all headings are required for purposes of navigation, and as observations are sometimes made on only two or three headings, it is important to determine B, C, D, E in order that the deviation on any desired heading may be computed.

In the case of observations on 16 equidistant headings, there will be 16 observation equations from which to compute the four coefficients by the method of least squares. In the formation of the normal equations the observation equations may be combined in such a way as to eliminate the constant term A and to leave only a single unknown in each normal equation, as shown in the sample computation given later on. As the declination is not known at sea and the final value of A is not determined until the end of the season's work, it is usually more convenient to make the analysis of that part of the deviations which does not involve A.

Since

$$\text{Compass error} - \text{Declination} = \text{Deviation}$$

and

$$\text{Mean compass error} - \text{Declination} = A$$

$$\text{Compass error} - \text{Mean compass error} = \text{Deviation} - A$$

For want of a better term this part of the deviation has been called "star deviation" and designated by an asterisk after the word, deviation*.

$$\text{Deviation}^* = \text{Deviation} - A = B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta.$$

The compass error is usually determined in one of three ways: (1) By observations of the sun; (2) by reciprocal bearings with a shore station; (3) by observing on a range of which the true bearing is known.

(1) The compass bearing of the sun is observed by means of an azimuth circle, and the true bearing is computed from the latitude of the place and the local mean time of observation. This requires, in addition to the latitude, a knowledge of the longitude and the correction of the chronometer on standard time. The computation is very much simplified by the use of United States Hydrographic Publication No. 71, or similar azimuth tables.

(2) For observations near shore it is sometimes more convenient to make use of the method of reciprocal bearings. An observer on shore measures the angle between the ship's binnacle and a reference mark, at the same moment that the observer on the ship measures the compass bearing of the shore station. If the true bearing of the reference

mark from the shore station is known, the true bearing of the shore station from the ship at the time of the compass observation may be computed. An older form of this method, and one which may be used when azimuth observations are impossible, is to mount a compass on shore and observe the compass bearing of the ship, the difference of the reciprocal bearings being the deviation for that particular heading, provided the shore station is free of local disturbance and the compass free of index error.

(3) In some harbors the true bearings of well defined range lines have been computed for the convenience of navigators and the compass error may be determined by observing the compass bearing of one of these ranges.

The forms of record and computation are shown in the following example. In this case the compass bearing of the sun was observed on 16 equidistant headings while swinging first with starboard helm and then with port helm, but only the starboard observations are reproduced.

Form 354.

OBSERVATION OF COMPASS DEVIATIONS.

Steamer, *Bache*. Standard compass No. 30367.
Date, July 10, 1909. Observer, W. C. Hodgkins.
Weather, clear. Sea, choppy. Wind, SSW.
Ship swung with starboard helm.

Ship's head by standard compass.	Time by hack watch No. 141.	Sun's bearing by standard compass.	Remarks.	
	h. m. s.	° ′		° ′
WSW.	5 49 40	N. 70 50 W.	Latitude	38 20
SW.	52 50	70 30	Longitude	76 22.3
SSW.	55 50	70 00		
S.	58 10	70 00	Chronometer comparison:	
SSE.	59 50	70 05		
SE.	6 02 05	69 10		h. m. s.
ESE.	05 42	67 30	Hack reads	5 10 19
E.	07 30	66 05	Chron. 3012	10 08 00
ENE.	10 10	65 00	Chron. corr'n	+ 54
NE.	12 30	64 25	G. M. T.	10 08 54
NNE.	15 10	64 20	E.	− 5 05
N.	17 00	66 00	G. A. T.	10 03 49
NNW.	19 40	67 00	Longitude	5 05 29
NW.	22 20	67 30	Local A. T.	4 58 20
WNW.	24 05	67 35	Hack reads	5 10 19
W.	26 10	67 05	Hack correction on local apparent time	− 11 59

Form 355.

Computation of Compass Deviations.

Steamer, *Bache.* Date, July 10, 1909.
Lat. 38° 20′ N., long. 76° 22′.3. Sun's declination, 22° 14′ N.
Ship swung with starboard helm.

Ship's head.	Local apparent time.	Sun's bearing by compass.	Sun's azimuth from tables.	Error of standard compass.	Deviation.*
	h. m. s.	° ′	° ′	° ′	° ′
N.	6 05 02	N. 66 00 W.	N. 71 31 W.	5 31 W.	0 32 W.
NNE.	6 03 12	64 20	71 46	7 26	2 27 W.
NE.	6 00 32	64 25	72 09	7 44	2 45 W.
ENE.	5 58 12	65 00	72 28	7 28	2 29 W.
E.	55 32	66 05	72 51	6 46	1 47 W.
ESE.	53 44	67 30	73 06	5 36	0 37 W.
SE.	50 07	69 10	73 36	4 26	0 33 E.
SSE.	47 52	70 05	73 55	3 50	1 09 E.
S.	46 12	70 00	74 09	4 09	0 50 E.
SSW.	43 52	70 00	74 28	4 28	0 31 E.
SW.	40 52	70 30	74 54	4 24	0 35 E.
WSW.	37 42	70 50	75 20	4 30	0 29 E.
W.	6 14 12	67 05	70 13	3 08	51 E.
WNW.	12 07	67 35	70 31	2 56	2 03 E.
NW.	10 22	67 30	70 46	3 16	1 43 E.
NNW.	07 42	67 00	71 08	4 08	0 51 E.
Means	5 56 42	67 42	72 41	4 59 W.	

Magnetic declination from shore observations, 5° 25′ W.

Form 356.

Analysis of Compass Deviations.*

Steamer, *Bache*. Date, July 10, 1909.

Ship's head.	Deviation.* Port.	Deviation.* Starb.	(1) Mean.	Ship's head.	Deviation.* Port.	Deviation.* Starb.	(2) Mean.		(3) (1)+(2)
	′	′	′		′	′	′		′
N.	− 53	− 32	− 42	S.	+ 39	+ 50	+ 44	*a*	+ 2
NNE.	− 142	− 147	− 144	SSW.	− 14	+ 31	+ 8	*b*	− 136
NE.	− 148	− 165	− 156	SW.	− 17	+ 35	+ 9	*c*	− 147
ENE.	− 105	− 149	− 127	WSW.	+ 65	+ 29	+ 47	*d*	− 80
E.	− 79	− 107	− 93	W.	+ 106	+ 111	+ 108	*e*	+ 5
ESE.	− 17	− 37	− 27	WNW.	+ 96	+ 123	+ 110	*f*	+ 83
SE.	+ 45	+ 33	+ 39	NW.	+ 114	+ 103	+ 108	*g*	+ 147
SSE.	+ 58	+ 69	+ 64	NNW.	+ 55	+ 51	+ 53	*h*	+ 117

Computation of B and C.					Computation of D and E.				
(4) (1)−(2)	(5)	(4)×(5)	(6)	(4)×(6)	(7) From (3)	(8)	(7)×(8)	(9)	(7)×(9)
′									
− 86	.000	00	1.000	− 86	*a*−*e*				
− 152	.383	− 58	.924	− 140	− 13	.000	00	1.000	− 13
− 165	.707	− 117	.707	− 117	*b*−*f*				
− 174	.924	− 161	.383	− 67	− 219	.707	− 155	.707	− 155
− 201	1.000	− 201	.000	00	*c*−*g*				
− 137	.924	− 127	− .383	+ 52	− 294	1.000	− 294	.000	00
− 69	.707	− 49	− .707	+ 49	*d*−*h*				
+ 11	.383	+ 4	− .924	− 10	− 197	.707	− 139	− .707	+ 139
	8B	− 709	8C	− 319		8D	− 588	8E	− 29
	B	− 89	C	− 40		D	− 74	E	− 4

N. B.—When observations are made on only 8 points, the divisors must be changed from 8 to 4.

Comparison of Observed and Computed Deviations.*

Deviation* = Deviation − A = B sin ζ + C cos ζ + D sin 2 ζ + E cos 2 ζ.

ζ is the compass azimuth of the ship's heading, counting from north around by east, south, and west to 360°.

Ship's head.	− 89′ B sin ζ	− 40′ C cos ζ	− 74′ D sin 2 ζ	− 4′ E cos 2 ζ	Deviation.*		C−O	
					Comp'd.	Obs'd.	v	v^2
	′	′	′	′	′	′		
N.	00	− 40	00	− 4	− 44	− 42	− 2	4
NNE.	− 34	− 37	− 52	− 3	− 126	− 144	+ 18	324
NE.	− 63	− 28	− 74	0	− 165	− 156	− 9	81
ENE.	− 82	− 15	− 52	+ 3	− 146	− 127	− 19	361
E.	− 89	00	00	+ 4	− 85	− 93	+ 8	64
ESE.	− 82	+ 15	+ 52	+ 3	− 12	− 27	+ 15	225
SE.	− 63	+ 28	+ 74	0	+ 39	+ 39	0	0
SSE.	− 34	+ 37	+ 52	− 3	+ 52	+ 64	− 12	144
S.	00	+ 40	00	− 4	+ 36	+ 44	− 8	64
SSW.	+ 34	+ 37	− 52	− 3	+ 16	+ 8	+ 8	64
SW.	+ 63	+ 28	− 74	0	+ 17	+ 9	+ 8	64
WSW.	+ 82	+ 15	− 52	+ 3	+ 48	+ 47	+ 1	1
W.	+ 89	00	00	+ 4	+ 93	+ 108	− 15	225
WNW.	+ 82	− 15	+ 52	+ 3	+ 122	+ 110	+ 12	144
NW.	+ 63	− 28	+ 74	0	+ 109	+ 108	+ 1	1
NNW.	+ 34	− 37	+ 52	− 3	+ 46	+ 53	− 7	49
							Σv^2	1815

Probable error of single observation, $r = \pm 8'$.

For 16 points, $r = \pm 0.195 \sqrt{\Sigma v^2}$. For 8 points, $r = \pm 0.337 \sqrt{\Sigma v^2}$.

Before and after the sun observations the observing timepiece was compared with the standard chronometer and its correction on local apparent time computed as shown. The mean of the two comparisons gave the correction − 11^m 58^s, and this was applied to the recorded times of observation to get the local apparent times of observation given in the second column of the form for "Computation of Compass Deviations."

Hydrographic Office Publication No. 71 gives the sun's azimuth at 10-minute intervals between sunrise and sunset for each degree of latitude from 61° N. to 61° S. and for each degree of declination of the sun. As three interpolations are in general required to get a desired azimuth, it expedites the computation of a series of observations to prepare from the azimuth tables an auxiliary table with which only a single interpolation will be necessary. In the example given the observations extended from 5^h 37^m to 6^h 47^m p. m., and the following table was prepared to cover that period for latitude 38° 20′ N. and sun's declination 22° 14′ N.

Azimuth of the Sun on July 10, 1909.

Declination. Latitude.	22° N. 38° N.	22° 14′ N. 38° N.	22° 14′ N. 39° N.	22° 14′ N. 38° 20′	Change per min.
h. m.	° ′	° ′	° ′	° ′	′
5 30	N. 76 30 W.	N. 76 18 W.	N. 76 38 W.	N. 76 25 W.	
40	75 07	74 55	75 13	75 01	8.4
50	73 44	73 32	73 48	73 37	8.4
6 00	72 20	72 08	72 23	72 13	8.4
10	70 56	70 45	70 57	70 49	8.4
20	69 31	69 20	69 30	69 23	8.6
30	68 06	67 55	68 02	67 57	8.6
40	66 39	66 28	66 34	66 30	8.7
50	65 11	65 00	65 04	65 01	8.9

A column has been added containing the values for latitude 28° N. and declination 22° N. taken directly from the azimuth tables. A comparison of these values with the corresponding ones in column 5 shows differences changing gradually from 5′ at the beginning to 10′ at the end. From this it will be seen that the desired azimuths may be obtained without the aid of an auxiliary table if the corrections to the table for the nearest even degree of latitude and declination be computed for the beginning and end of the series of observations. As it is the usual practice to combine swings with port and starboard helms, it will be sufficiently accurate for practical purposes to determine the correction for the middle of each swing and assume that it is constant throughout the swing.

The difference between the observed compass bearing of the sun and its computed true bearing is the error of the compass for that heading. By subtracting the mean compass error from the error for each heading the corresponding deviations * (star deviations) are found, provided observations have been made on each of 8 or 16 equidistant headings. It sometimes happens that the observation on one heading is prevented by the mast or funnel coming in line with the sun. In such case the missing compass error must be supplied by interpolation before taking the mean. This can usually be done with sufficient accuracy by comparison with the swing in the opposite direction. If the observations on several headings have been prevented by clouds, graphical interpolation should be resorted to, plotting the observed compass errors and drawing a smooth curve to represent them.

The analysis of the compass deviations * requires little explanation, as the order of computation is indicated by the headings of the form. For two headings ζ and $(180° + \zeta)$ the observation equations would be:

$$\text{Deviation}^* (\zeta) = B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

and

$$\text{Deviation}^* (180° + \zeta) = -B \sin \zeta - C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

Hence

$$\text{Deviation}^* (\zeta) - \text{Deviation}^* (180° + \zeta) = \varDelta_1 = 2B \sin \zeta + 2C \cos \zeta$$

and

$$\text{Deviation}^* (\zeta) + \text{Deviation}^* (180° + \zeta) = \varDelta_2 = 2D \sin 2\zeta + 2E \cos 2\zeta$$

It will be seen that the quantities on the same line in columns headed (1) and (2) are in each case the deviations * for two headings 180° apart, and hence the quantities in column (3) involve only the factors D and E (quadrantal deviation) and those in column (4) involve only B and C (semicircular deviation).

From observation equations of the form:

$$\varDelta_1 = 2B \sin \zeta + 2C \cos \zeta$$

the values of B and C are obtained by the method of least squares from the normal equations:

$$\Sigma \varDelta_1 \sin \zeta = 2B \Sigma \sin^2 \zeta + 2C \Sigma \sin \zeta \cos \zeta$$

$$\Sigma \varDelta_1 \cos \zeta = 2B \Sigma \sin \zeta \cos \zeta + 2C \Sigma \cos^2 \zeta$$

The values of $\sin \zeta$ and $\cos \zeta$ for angles corresponding to the equidistant headings N, NNE, ---- SSE, are given in columns (5) and (6). It will be seen that $\Sigma \sin^2 \zeta = 4$, $\Sigma \sin \zeta \cos \zeta = 0$ and $\Sigma \cos^2 \zeta = 4$. Hence for the case of observations on 16 equidistant headings the normal equations become:

$$\Sigma \varDelta_1 \sin \zeta = 8B \qquad \Sigma \varDelta_1 \cos \zeta = 8C$$

and the computation is made in the simple manner indicated on the form. For observations on 8 equidistant headings only the values of $\sin \zeta$ and $\cos \zeta$ given on the first, third, fifth, and seventh lines will be involved and the normal equations will be:

$$\Sigma \varDelta_1 \sin \zeta = 4B \qquad \Sigma \varDelta_1 \cos \zeta = 4C$$

In the publication of the various Hydrographic Offices treating of the compass and its deviations, tables are given to facilitate the computation of $\varDelta_1 \sin \zeta$ and $\varDelta_1 \cos \zeta$, where the deviations are expressed in degrees and minutes. For the small deviations involved in observations on the ships of the Coast and Geodetic Survey it will be found convenient to convert the deviations * to minutes and use Table XIII given at the end of this publication.

It has been shown above that the sum of the observation equations for two headings 180° apart would be:

$$\varDelta_2 = 2D \sin 2\zeta + 2E \cos 2\zeta$$

For the two headings 90° from the first two the quantities in the second member would be the same, but the signs would be changed. The difference of the two equations would give:

$$\Delta_3 = 4\ D \sin 2\zeta + 4\ E \cos 2\zeta$$

The values of Δ_3 are given in the column headed (7), obtained from column (3) in the manner indicated. The corresponding normal equations are:

$$\Sigma\Delta_3 \sin 2\zeta = 8\ D \qquad \text{and} \qquad \Sigma\Delta_3 \cos 2\zeta = 8\ E$$

for observations on 16 headings,

and $$\Sigma\Delta_3 \sin 2\zeta = 4\ D \qquad \text{and} \qquad \Sigma\Delta_3 \cos 2\zeta = 4\ E$$

for observations on 8 headings. The quantities in columns (8) and (9) are the sines and cosines of 0°, 45°, 90°, and 135°, respectively.

The analysis of the observations in the example gives:

$$\text{Deviation}^* = -89' \sin \zeta - 40' \cos \zeta - 74' \sin 2\zeta - 4' \cos 2\zeta$$

from which the deviation * on any heading, ζ, can be computed. As a test of the accuracy of the observations a comparison should be made between the deviations * derived from the observations and those computed from the formula. For observations on 16 headings the probable error of a single observation (mean of swings) is given by the formula:

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{16-4}} = 0.195\sqrt{\Sigma v^2}$$

Where observations are made on only 8 headings,

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{8-4}} = 0.337\sqrt{\Sigma v^2}$$

In the example $r = \pm 8'$. For observations under favorable conditions that represents about an average value.

For navigational purposes the complete deviations are required. They may be obtained by adding the constant part of the deviation, A, to the deviations.* As already pointed out, A is determined from swings near shore where the declination is known at least approximately. As the declination where the ship is swung is in general not exactly the same as at the point on shore where observations are made, it is desirable to combine the results obtained at a number of places to get a mean value of A. While this constant part of the compass deviation is no doubt partly due to unsymmetrical distribution of the ship magnetism with respect to the compass, the greater part is to be ascribed to imperfections in the compass and the azimuth circle, corresponding to an index error.

Since the deviation is the resultant effect of the forces exerted on the compass needle by the ship's magnetism and the earth's magnetism, it follows that any change in the ratio of those two forces will produce a change in the deviation. The quadrantal deviation is due to the magnetism induced in horizontal soft iron and therefore varies directly as H varies, and for a particular heading the ratio of the two does not change when the ship goes from place to place. Hence the coefficients D and E should be constant.

The semicircular deviation is due partly to the subpermanent magnetism of the ship and partly to induced magnetism in vertical soft iron. The former is constant, or nearly so, and therefore produces an effect on the compass needle which is inversely proportional to H. The induced magnetism in vertical soft iron is proportional to the vertical force or $H \tan I$, and its effect on the compass needle is therefore proportional to $\tan I$. As H decreases and I increases in going from the magnetic equator to the magnetic poles, it follows that in the northern hemisphere B and C should become greater as the ship goes farther north and vice versa.

DIP AND TOTAL INTENSITY.

On several of the ships of the Coast and Geodetic Survey dip and total intensity are determined by means of a Lloyd-Creak dip circle mounted on a gimbal stand as shown in Figure 9. The balance of the instrument is secured by a counter-poise at the back, and its stability is regulated by a heavy ball threaded onto a rod extending below the gimbal rings. The instrument is leveled in the same manner as when mounted on a tripod on land.

Experience with the original form of Lloyd-Creak dip circle showed that within 30° or 40° of the magnetic equator it was impossible to observe deflections, as the earth's total intensity became too small to offer sufficient resistance to the force exerted by the deflecting needle, and the suspended needle would not come to rest at right angles to the deflector. To remedy this defect several dip circles of this type have been modified in the instrument shop of the Coast and Geodetic Survey so as to increase the distance between the two needles when making deflection observations. The loaded needle, when in use as a deflector, is mounted in an aluminum case which fits in a frame between the reading microscopes, as shown in Figure 9. The needle is mounted to one side of the center of the case, so that the deflections may be made at two deflection distances by reversing the position of the case in its supporting frame. Originally the needles were 7.3 cm. apart during deflections. With the new arrangement the distances are 7.9 and 9.4 cm., respectively, and it is possible to observe, at least at the longer distance, in any part of the globe. In the instrument shown in the figure a small telescope was added, so that astronomical observations on land can be made if desired.

Dip observations may be made with the regular dip needles in the same manner and with nearly the same facility as on land. The only difference is that the ivory scraper must be used continuously, and when the needle swings through a large arc on account of the motion of the ship, the extremes of the swing must be read and recorded instead of attempting to estimate the middle. As it is usual to make observations on 8 or 16 equidistant headings in order to eliminate the varying effect of the ship's magnetism, a complete determination of dip and intensity on each heading would require too much time,

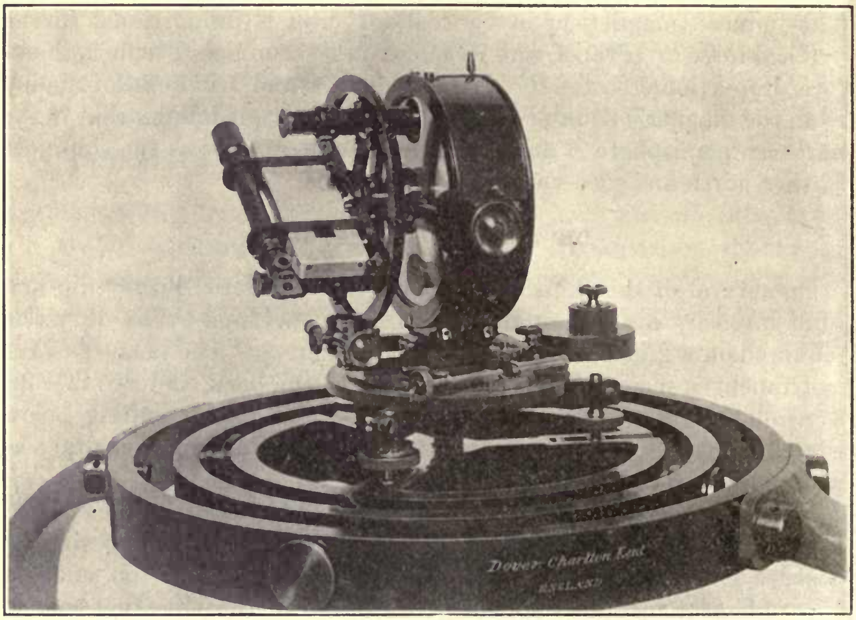

FIG. 9.—Lloyd-Creak pattern dip circle.

and the following scheme of observations has been adopted with satisfactory results.

(1) Deflection observations are made while swinging ship in one direction and dip with loaded needle while swinging in the opposite direction. The combination of these observations will give a value of total intensity for each heading, and from each observation of deflections a value of dip may be obtained, as explained on page 83.

(2) On each heading observations are made in only one position of needle and circle, so as to require not much more time than the compass observations, which are usually going on at the same time. As the gimbal stand is set so that when the horizontal circle of the dip circle reads zero the needle swings in the plane of the fore-and-aft line

of the ship, the instrument may be placed in the magnetic meridian with sufficient accuracy by means of the heading of the ship as shown by the standard compass. For observations on 16 headings the arrangement would be as follows:

Heading.	Needle face.	Ver. circle.	Hor. circle setting.
			° ′
N.	E.	E.	360 00
NNE.	E.	E.	337 30
NE.	E.	E.	315 00
ENE.	E.	E.	292 30
E.	W.	W.	90 00
ESE.	W.	W.	67 30
SE.	W.	W.	45 00
SSE.	W.	W.	22 30
S.	E.	W.	360 00
SSW.	E.	W.	337 30
SW.	E.	W.	315 00
WSW.	E.	W.	292 30
W.	W.	E.	90 00
WNW.	W.	E.	67 30
NW.	W.	E.	45 00
NNW.	W.	E.	22 30

(3) Two readings of each end of the needle are made for each position. In case the needle is so nearly horizontal that only one end can be read, four readings are made of that end.

(4) The times of beginning and ending of observations for each group of four headings and the corresponding temperatures are recorded, also conditions of weather, sea, etc.

The variation in the values of I, I', and u for the four different positions of circle and needle can be determined from the shore observations, and the values obtained on shipboard should be corrected accordingly to reduce to the mean of the four positions, in case the corrections amount to as much as $10'$; in any case the values of dip must be corrected to reduce to the standard dip instrument. Sample observations of the two classes and the corresponding computations, and a summary of the results of a complete swing, are given below.

Form 391.

Magnetic Observations on Board C. and G. S. S. Bache.

Loaded Dip for Total Intensity.

Date, July 10, 1909. Latitude, 38° 20′. Longitude, 76° 22′.
Dip circle No. 35. Needle No. 4. Weight No. 6.

Ship's head N. Hor. circle 360°. Ver. circle E. Needle face E.		Ship's head NNE. Hor. circle 337½°. Ver. circle E. Needle face E.		Ship's head NE. Hor. circle 315°. Ver. circle E. Needle face E.		Ship's head ENE. Hor. circle 292½°. Ver. circle E. Needle face E.	
S.	N.	S.	N.	S.	N.	S.	N.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
345 00	164 40	346 20	166 00	348 50	168 40	350 50	170 30
344 10	164 30	346 00	166 10	348 10	168 30	351 00	170 50
344 35	164 35	346 10	166 05	348 30	68 35	350 55	170 40
I′ −15 25		−13 52		−11 28		−9 12	

Form 390.

Magnetic Observations on Board C. and G. S. S. Bache.

Deflections for Total Intensity and Dip.

Date, July 10, 1909. Latitude, 38° 20′. Longitude, 76° 22′.
Dip circle No. 35. Needle No. 4 deflecting, No. 3 suspended.

Ship's head NE. Ver. circle E.		Hor. circle 315°. Needle face E.		Ship's head NNE. Ver. circle E.		Hor. circle 337½°. Needle face E.	
D.		R.		R.		D.	
N.	S.	N.	S.	N.	S.	N.	S.
° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
228 40	48 20	285 10	105 20	287 10	107 30	229 40	49 50
30	30	20	20	20	40	40	50
228 35	48 25	285 15	105 20	287 15	107 35	229 40	49 50
48 30		105 17.5		107 25		49 45	
153 47.5		56 47.5		157 10		57 40	
I 76 54		*u* 28 24		*I* 78 35		*u* 28 50	

Form 392.

Computation of Total Intensity.

From observations on board C. and G. S. S. *Bache.*

Formulas: $u'=I-I'$ and $F=C\sqrt{\cos I' . \csc u . \csc u'}$

Date, July 10, 1909. Computer, H. M. Armstrong

Ship's head.	N.	NNE.	NE.	ENE.
	° ′	° ′	° ′	° ′
I′	−15 25	−13 52	−11 28	− 9 12
u	29 03	28 59	28 33	27 49
u′	94 16	92 04	87 59	83 39
log cos *I′*	9. 98409	9. 98715	9. 99124	9. 99438
" csc *u*	0. 31375	0. 31466	0. 32064	0. 33101
" csc *u′*	0. 00121	0. 00028	0. 00027	0. 00267
Sum	0. 29905	0. 30209	0. 31215	0. 32806
Half sum	0. 14952	0. 15104	0. 15608	0. 16403
log *C*	9. 62425	9. 62425	9. 62425	9. 62425
" *F*	9. 77377	9. 77529	9. 78033	9. 78828
F	.5940	.5961	.6030	.6142

Summary of Results.

Steamer *Bache,* July 10, 1909.

Head.	*I* obs'd.	*I* corr'd.	*I′*	*u* obs'd.	*u* corr'd.	*u′*	*F*
	° ′	° ′	° ′	° ′	° ′	° ′	*C. G. S.*
N.	79 14	78 51	−15 25	28 54	29 03	94 16	0.5940
NNE.	78 35	78 12	−13 52	28 50	28 59	92 04	.5961
NE.	76 54	76 31	−11 28	28 24	28 33	87 59	.6030
ENE.	74 50	74 27	− 9 12	27 40	27 49	83 39	.6142
E.	71 01	71 42	− 5 20	27 24	27 32	77 02	.6258
ESE.	68 11	68 52	− 2 42	26 19	26 27	71 32	.6473
SE.	65 52	66 33	− 0 58	25 28	25 36	67 31	.6662
SSE.	64 21	65 02	0 00	25 04	25 12	65 02	.6776
S.	64 48	64 29	+ 0 18	25 00	24 50	64 11	.6846
SSW.	65 50	65 31	+ 0 02	25 20	25 10	65 29	.6768
SW.	68 18	67 59	− 0 48	26 02	25 52	68 47	.6601
WSW.	71 28	71 09	− 2 40	26 42	26 32	73 49	.6424
W.	74 09	74 43	− 4 55	27 56	27 49	79 38	.6202
WNW.	76 41	77 15	− 9 12	28 34	28 27	86 27	.6066
NW.	78 15	78 49	−13 02	28 55	28 48	91 51	.5988
NNW.	78 49	79 23	−15 05	29 04	28 57	94 28	.5955
Mean.	72 20	72 28	− 6 31	27 14	27 14	78 59	.6318

In the case of this dip circle, No. 35, it was found necessary to apply the following corrections to *I* and *u* on the basis of the results of observations at several shore stations:

	I	*u*
Circle east, needle face east	−23′	+ 9′
Circle west, needle face west	+41	+ 8
Circle west, needle face east	−19	−10
Circle east, needle face west	+34	− 7

It will be observed that the values of dip and total intensity show a large range in the course of the swing. It will be found also by comparison with shore observations in the vicinity that the mean values of dip and intensity on board differ by considerable amounts from the shore results. These differences do not remain constant, however, when the ship goes from place to place. The deviations in dip and total intensity may be derived and analyzed in a manner similar to that given for the compass deviations. As it is seldom, however, that dip-circle observations are made at sea except when swinging ship, it is usually sufficient to obtain by interpolation from the swings near shore the corrections required by the mean values of dip and total intensity for the swings at sea.

For the limited range of dip usually covered in a season's work and with swings near shore in the highest and lowest latitudes reached, a satisfactory approximation is obtained by assuming that the changes in the corrections are proportional to the changes in dip.

SPECIAL DIRECTIONS.

In order to obtain the best results from observations on shipboard especial attention should be paid to the following points:

(*1*) *Avoid as far as possible any change in the condition of the intensity needles. Keep them clean and free from rust and take especial care to protect the pivots from injury. When removing a needle after observing be sure that the point does not catch on the edge of the graduated circle.*

(*2*) *Have the ship as nearly as possible in the same condition as regards location of boats, anchors, chains, etc., for the swings near shore as for those at sea.*

(*3*) *Make the compass observations when the sun is not more than 30° high, if possible. Steady the ship on a heading for a minute or two before reading the sun's bearing. In handling the azimuth circle, be careful to have the compass bowl swinging free at the moment of observation.*

(*4*) *When there is much motion to the ship, select a moment for taking a reading when she is nearly on an even keel. In the case of the dip circle, select a vibration of the needle which appears symmetrical.*

DIRECTIONS FOR OPERATING A MAGNETIC OBSERVATORY.

BUILDINGS.

For the operation of a magnetic observatory there are required a *variation building* in which the variation instruments are mounted, and an *absolute building* in which the absolute observations are made. In their construction scrupulous care is exercised to exclude all material that might possibly affect the magnets, and in their subsequent use the same care must be exercised. No article of magnetic material should be carried into the buildings unless absolutely needed, and no person should be allowed to enter the variation building until he has divested himself of all such articles, as knife, watch, keys, etc. The sole of a shoe or the brim of a hat often contains a piece of steel sufficient to disturb the sensitive variation instruments. Such other buildings as may be needed are usually placed far enough away to have no effect on the magnets. The variation building is designed with a view to reduce to a small limit the range of temperature inside. Those of the Coast and Geodetic Survey are all above ground and built of wood, the amount of insulation varying with the range of temperature to be overcome.

VARIATION INSTRUMENTS.

The variations in declination, horizontal intensity, and vertical intensity are recorded photographically by means of a magnetograph, consisting of a recording apparatus and three variometers. Light from a lamp is reflected from a mirror attached to the magnet of a variometer and traces an irregular line (curve) on a sheet of photographic paper (magnetogram) wrapped around a revolving drum of the recording apparatus. The reflection from a fixed mirror traces a straight line (base line) on the magnetogram, and the variation in the distance between the curve and the base line (ordinate) is a measure of the variation in the direction of the suspended magnet produced by the variation in the earth's magnetism.

The D variometer is mounted with its magnet in the magnetic meridian and the direction of the magnet changes as the magnetic declination changes. In the H variometer the magnet is suspended in the magnetic prime vertical and a change in its direction corresponds to a change in the horizontal intensity. In the Z variometer the magnet rotates about a horizontal axis, like a dip needle, but is adjusted to lie approximately in the horizontal plane, so that a variation in its inclination to the horizon corresponds to a variation in the vertical intensity.

Each of the five observatories of the Coast and Geodetic Survey is equipped with a magnetograph of the Eschenhagen type, in which very small magnets are used, so that it is possible to have the variometers quite near to each other without appreciable interaction. They are mounted in a row, magnetically east and west, as shown in Figure 10, and all three record on the same magnetogram, the up and down motion of the *Z* magnet being converted to horizontal motion on the magnetogram by means of a prism. There is also a thermograph attached to the vertical intensity variometer, which records photographically the variations of temperature.

When the *D* variometer is east of the recording apparatus, a motion of the north end of the magnet to the east (increasing declination) causes the registering spot of light to move to the north on the recording drum. When it is west of the recording apparatus the motion of the spot of light is toward the south. In the case of the horizontal intensity variometer, when the north end of the

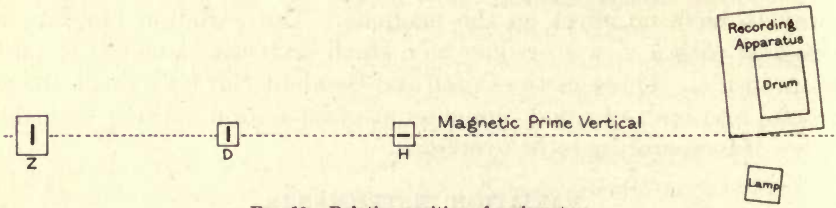

FIG. 10.—Relative position of variometers.

magnet points toward the recording apparatus, an increase of horizontal intensity causes the spot of light to move toward the north on the drum, no matter whether the magnet is east or west of the recording apparatus. The vertical intensity variometer is mounted with the north end of the magnet to the north, and an increase of vertical intensity causes the spot of light to move toward the south on the drum. In the *D* and *H* variometers the magnet is suspended by means of a quartz fiber. To the lower end of the fiber is attached a light frame supporting two mirrors, the vertical plane surfaces of which make a slight angle with each other. They must be so adjusted that the distance between the two reflected rays of light, where they strike the recording cylinder, is slightly less than the width of the magnetogram. The magnet hangs in a stirrup attached to the lower part of the mirror frame and is inclosed by a cylindrical copper damping box. As the diameter of the box is only slightly greater than the length of the magnet and as the hole in the top is not much larger than the shank of the stirrup, it is necessary in adjusting the instrument to see that the shank is centered in the hole in order to insure freedom of motion of the magnet, and the observer should examine the variometers from time to time to be sure that no change of level has occurred. The moment of inertia of the magnet system is so small that only a very small resistance is required to appreciably affect the motion of the magnet. An accumulation of mold or dust, or the web of a minute spider, such as occasionally finds its way through the openings of the variometer, is often sufficient. The appearance of the curve will usually indicate the presence of an obstruction, and it may be necessary to remove the damping boxes and clean them unless it is found that change of level eliminates the trouble. It sometimes happens that a gradual slipping of the joint between fiber and mirror frame causes a lowering of the magnet in its damping box until it finally touches the bottom.

The magnet of the *H* variometer is held in its position at right angles to the magnetic meridian principally by the torsion of the supporting fiber, but partly also by control magnets placed below the suspended magnet with their axes horizontal. Their primary object, however, is to regulate the sensitiveness of the variometer.

In the *Z* variometer there are two magnets supported by a light frame, like the beam of a balance, on the under side of which are two conical steel points which rest in cup-shaped depressions in agate surfaces. The steel points are so adjusted that the line joining them is at right angles to the magnetic axis of the magnet system. The horizontality of the magnet is secured partly by an adjustable weight threaded onto a horizontal rod, and partly by a control magnet directly under the center of the magnet, which partly counteracts the effect of *Z*. The stability is regulated by a weight threaded onto a vertical rod, so that the center of gravity of the magnet system may be raised or lowered, as desired. Great care must be exercised in working about this variometer, as a slight jar is sometimes sufficient to change the balance of the magnet system or even throw it out of balance altogether. This applies especially to the deflection observations for the determination of scale value. When the magnet system has become displaced by a jar or by an unusually large deflection, as during a magnetic storm, it may generally be put back in adjustment by lifting and lowering two or three times by means of the lifter. If further adjustment is required, it can usually be done by raising or lowering the control magnet, unless a change of sensitiveness is required.

Under unfavorable conditions the steel bearing points may become rusted or otherwise blunted, in which case the magnet system will not move freely and the curve will appear abnormally smooth. Extra bearing points are provided for replacing those which may become defective.

The distance between the *D* and *Z* variometers is so short that a change in the position of the *Z* control magnet produces a slight change in the position of the D magnet, and the amount of this change must be determined when the Z variometer is readjusted.

The thermograph consists of a Bourdon tube filled with mercury, to the free end of which is attached a mirror.

The drum of the recording apparatus is usually made to revolve once in 24 hours. At hourly intervals a system of shutters is raised to a vertical position by means of a cam attached to a gear wheel and after the lapse of a minute or two allowed to drop back into a horizontal position. The shutters are so adjusted that when raised they prevent the light from the fixed mirrors reaching the magnetogram and thus produce a short break in each base line.

By a change of gearing the drum may be made to revolve once in two hours and the time breaks then occur every five minutes. It is then necessary to open wider the slit of the lamp, in order to secure sharp lines on the photographic paper.

CONVERSION TO ABSOLUTE VALUES.

In order to determine the absolute value of D, H, or Z at any moment from the continuous photographic record of the variometer it is necessary to know: (1) The base-line value, that is, the absolute value when the curve and base line coincide; (2) the scale value, or value of 1 millimeter of ordinate expressed in absolute units (minutes for D, gammas or units of the fifth decimal in the C. G. S. system for H and Z); (3) in the case of H and Z, the temperature coefficient, or the effect upon the ordinate of a change of 1° in temperature.

In the H variometer, the position of the magnet is the resultant effect of the force of torsion, the force acting between the suspended magnet and the control magnets, and the force exerted upon the suspended magnet by the horizontal component of the earth's field. A change in the magnetic moment of the suspended magnet due to change of temperature will change the magnetic force acting and hence change the position of equilibrium, irrespective of any change in H.

Similar conditions exist in the Z variometer and in addition the moment of the balancing weight changes with temperature on account of change in the length of the supporting rod.

Let d, h, z = the ordinates in millimeters at the temperature t, increasing ordinate corresponding to increasing D, H, and Z,

ε_d, ε_h, ε_z = scale values of D, H, and Z, respectively,

B_d, B_h, B_z = base-line values for D, H, and Z (in the case of H and Z, reduced to a standard temperature t_o,

q_h and q_z = temperature coefficients of the H and Z variometers,

Then

$$D = B_d + d.\varepsilon_d$$
$$H = B_h + h.\varepsilon_h + q_h(t - t_o)$$
$$Z = B_z + z.\varepsilon_z + q_z(t - t_o)$$

BASE-LINE VALUES.

For the determination of the base-line values absolute observations are made at least once a week. From an inspection of the above formulas it will be seen that if the ordinates d, h, z be read for the times at which absolute observations have been made, the base-line values may be computed, provided the scale values and temperature coefficients are known. The absolute value of vertical intensity must be computed, however, from the observed values of H and I. It is in general not feasible to make simultaneous observations of H and I, but the value of H at the time of the dip observations may be determined from the record of the variometer after the H base-line value has been computed.

The absolute observations are made in the manner already explained, but greater care must be exercised in the operations and a greater degree of accuracy is to be expected than is the case in work in the field. Absolute accuracy is impossible, however, and the base-line values resulting from a series of observations will show more or less variation, whereas they should be constant provided there has been no change in the adjustment of the variometers. It has been found in some cases that even when there has been no readjustment of the variometer the base-line values show a progressive change. This may be due partly to gradual change of the relative positions of suspension fiber and stirrup, partly to the fact that the magnets suffer gradual loss of magnetism with age and, in the case of H and Z where the range of temperature has been large, partly to error in the adopted value of temperature coefficient. It is possible also that the torsion of the quartz fibers may change somewhat with age. Hence in determining what base-line values to adopt it is necessary to adjust the observed values, having due regard to this progressive change. For any particular set of instruments it must be found out by experience how closely the adopted values should correspond to those resulting from observation. In the case of D and H the mean of the values determined in the course of a month is usually almost free of error of observation and may be used as a basis for determining the gradual change from month to month. In the case of Z the error of observation is somewhat greater. An abrupt and continued change in the base-line values when there has been no readjustment of the variometer would demand a careful examination of the absolute instrument to make sure that there is no systematic error in the absolute observations due to lack of adjustment. It is desirable to make absolute observations at different times of the day, so that the computation of base-line values will involve ordinates of different amounts. The form of computation of H and Z base-line values is shown in the following examples, in which the adopted standard temperature is 10° C.

Form 358.

Horizontal Intensity Base Line.

Sitka, Alaska, Magnetic Observatory.
Magnetograph No. 6. Magnetometer No. 37.

Date.	Feb. 5.		Feb. 11.		Feb. 17.		Feb. 20.	
1908.	L. M. T.	Scaling.	L. M. T.	Scaling.	L. M. T.	Scaling.	L. M. T.	Scaling.
	h. m.	*mm.*	*h. m.*	*mm.*	*h. m.*	*mm.*	*h. m.*	*mm.*
Oscillations.	10 21	73.4	14 40	71.7	9 17	73.6	10 05	68.3
	24	79.4	43	69.3	20	73.7	08	68.1
	26	74.4	46	66.7	22	73.6	10	67.7
	29	76.6	48	67.5	25	73.4	13	67.4
Deflections.	10 41	73.8	15 00	68.3	9 36	72.0	10 28	66.5
	47	74.4	06	68.9	41	71.4	33	67.6
	52	74.4	11	69.3	46	71.4	39	66.0
	58	72.7	17	69.6	51	71.3	44	65.4
Deflections.	11 07	72.9	15 21	70.3	9 56	71.2	10 48	65.3
	12	71.8	26	70.1	10 01	70.1	53	64.9
	17	71.6	32	70.1	06	69.8	58	64.6
	22	69.4	37	69.1	11	69.4	11 04	64.5
Oscillations.	11 29	65.3	15 45	69.5	10 19	68.3	11 12	63.5
	32	63.3	48	69.3	21	68.5	15	63.5
	34	61.3	50	70.0	24	68.7	18	63.4
	37	59.3	53	69.5	27	68.6	20	63.4
Mean		70.9		69.3		70.9		65.6
		γ		γ		γ		γ
ε_h		2.79		2.79		2.79		2.79
t and h_t	3°.7	198	5°.9	193	5°.7	198	6°.5	183
Δh		−46		−30		−31		−26
h_t		152		163		167		157
H		15536		15552		15555		15537
Base line		15384		15389		15388		15380

Form 359.

Vertical Intensity Base Line.

Sitka Magnetic Observatory.
Magnetograph No. 6. Earth inductor No. 2.

Date.	Feb. 5.			Feb. 11.		
1908.	Local mean time.	Scalings. H.	Scalings. Z.	Local mean time.	Scalings. H.	Scalings. Z.
	h. m.	mm.	mm.	h. m.	mm.	mm.
	9 29	77.2	−88.1	16 25	71.3	−80.6
	31	77.6	88.1	27	70.7	80.9
	34	77.4	87.7	30	72.1	81.2
	36	77.4	87.5	32	71.3	81.3
	39	79.4	87.4	34	72.8	80.7
	41	79.4	87.3	36	73.3	80.5
	44	78.2	86.7	39	73.3	80.6
Means	(q=7.3γ)	78.1	−87.5		72.1	−80.8
ε_h and ε_z		2.79	4.69		2.79	4.69
	°		γ	°		γ
t and h_t	3.8		218	5.9		201
Δt and Δh	6.2		−45	4.1		−30
h			173			171
H base line	at 10°		15386			15386
H			15559			15557
I			74° 36′.4			74° 37′.0
log H			4.19198			4.19193
log tan I			0.56016			0.56046
log Z			4.75214			4.75239
	(q=−1.0γ)		γ			γ
Z			56512			56544
	°			°		
t and z_t	3.8		−410	5.9		−379
Δt and Δz	6.2		+6	4.1		+4
z			−404			−375
Z base line	at 10°		56916			56919

SCALE VALUES.

From the formulas on page 106 it would appear that the scale value of a variometer might be determined by making absolute observations at different times and comparing the change in the observed values with the change in ordinate. In practice, however, the uncertainty in the absolute observations is generally too great to secure satisfactory results in this way.

The scale value of the D variometer depends directly upon the distance between the movable mirror of the magnet and the paper on the drum, and is found by the formula—

$$\varepsilon_d = \frac{\text{ctn}\,1'}{2R}\left(\frac{f}{f-h}\right) = \frac{3437.75}{2R}\left(\frac{f}{f-h}\right)$$

in which ε_d is the angular motion of the magnet corresponding to 1 mm on the magnetogram, R is the distance from the magnetogram to the face of the movable mirror increased by two-thirds the thickness of the mirror, and h is the angle through which the magnet is turned when the torsion head is turned through the angle f. The lens of the Eschenhagen D variometer is usually focussed for a scale value of 1 mm. = 1′. For this convenient scale value—

$$R = \overset{mm}{1718.9}\left(\frac{f}{f-h}\right).$$

There is so little variation in the torsion of a quartz fiber that after its torsion factor has once been determined it is unnecessary to repeat the operation, although it is usual to do so from time to time as a check. When a new fiber is inserted, a determination of its torsion factor is of course required.

In the case of the H variometer, the scale value depends upon the rigidity with which the magnet is held in its position at right angles to the magnetic meridian. In the older form of suspension (bi-filar) the desired sensitiveness is secured by varying the distance between the upper ends of the two supporting fibers. Where quartz fiber suspension is used, it is not practicable to regulate the sensitiveness by the size of the fiber, but the desired result is obtained by means of control magnets, which serve to increase or decrease the force to be balanced by the torsion of the fiber.

The scale value is found by comparing the amounts by which an auxiliary magnet deflects the magnets of the D and H variometers, when similarly placed with regard to them. The series of observations is begun by deflecting the D magnet by placing the deflector to the east and west (in the magnetic prime vertical), north end east or west. Then the H magnet is deflected by placing the deflector to the north and south, north end north or south. This is followed by a second set of D deflections. Unless the torsion of the D fiber is known from previous observations, a set of observations to determine that factor is also required. It is important to make deflections at two distances as a check on the accuracy of the work. Such distances should be selected as will give deflections of considerable magnitude without throwing the spot of light beyond the limits of the magnetogram. Care must be taken to use the same deflection distances on both variometers and to have the deflector in the same horizontal plane with the deflected magnet.

It is evident that the effect of the deflector on the H magnet corresponds to an increase or decrease of the horizontal intensity by an amount equal to the intensity of the field of the deflector at the selected distance. The amount by which the D magnet is deflected depends upon the relative intensity of the earth's field and that of the deflector at the selected distance.

Let u = the number of millimeters which the D spot is deflected.
U = the angle through which the D magnet is deflected.
u' = the number of millimeters which the H spot is deflected.
$\varepsilon_h = H$ scale value, i. e., the change in H, expressed in gammas, corresponding to a change in ordinate of 1 mm.
W = field intensity of the deflector at the selected distance.

In the case of the H deflections:

$$\varepsilon_h . u' = W$$

In the case of the D deflections:

$$W = H \tan U = H U \tan 1' = \frac{Hu \tan 1'}{2R \tan 1'}\left(\frac{f}{f-h}\right) = \frac{Hu}{2R}\left(\frac{f}{f-h}\right)$$

Since the deflection angle is always small its tangent may be taken as proportional to the angle and we have just seen that the D scale value is

$$\frac{u}{2R \tan 1'}\left(\frac{f}{f-h}\right)$$

Hence

$$\varepsilon_h . u' = \frac{Hu}{2R}\left(\frac{f}{f-h}\right) \quad \text{and} \quad \varepsilon_h = \frac{2u}{2u'} \cdot \frac{H}{2R}\left(\frac{f}{f-h}\right)$$

As the deflection observations give directly the values of $2u$ and $2u'$, it is more convenient to introduce a 2 in both numerator and denominator of the formula and use it in the form given. It has already been pointed out that there is very little variation in the torsion coefficient of a quartz fiber. The distance between the D mirror and the drum is constant so long as there is no readjustment and H may be assumed as constant for a year without introducing a material error in the scale value computation. Hence, for that period the factor $\frac{H}{2R} \cdot \frac{f}{f-h}$ may be regarded as constant and the formula becomes simply

$$\varepsilon_h = \frac{2u}{2u'}\left(\text{constant}\right)$$

The form of observation and computation is shown in the following example. It will be noticed that the two results from deflections at different distances do not agree exactly. A comparison of the H ordinates for times when the deflector was not in use with the deflected ordinates shows that the deflections were unsymmetrical, the amount of deflection being greater when the ordinate was decreased than when it was increased. It has been found that under certain conditions of adjustment, as in this case, the scale value varies with the position of the magnet and may be represented analytically as a linear function of the ordinate.

$$\varepsilon_h = A + Bh$$

A comparison of the different determinations at Cheltenham in 1909 gives the value $B = .004$. The mean of the scalings for the short deflection distance is 13.0 mm. and for the long distance 18.3 mm., and the resulting scale values should differ by about 0.02γ, as they do. When adjusting the variometer, the control magnets should be so placed that the deflections are symmetrical or nearly so.

It sometimes happens during deflections that the regular spot of light is thrown off the magnetogram and a record is made by the reserve spot. In such cases the two spots must be made to record at the same time at the close of the observations, so that the distance between them may be measured.

Form 373.

H Scale Value.

Date, May 26, 1909. Magnetograph No. 5. Observer, J. E. Burbank.

Instrument.	Magnet.	N. end.	I. Distance=26 cm. No.	I. Scaling.	I. Diff.	II. Distance=31 cm. No.	II. Scaling.	II. Diff.	Remarks.
				mm.	mm.		mm.	mm.	
D	E	E	1	38.0	40.0	4	46.1	23.9	
	E	W	2	78.0		3	70.0		
	W	W	8	77.9	39.8	5	69.8	23.4	
	W	E	7	38.1		6	46.4		
H	N	N	1	91.1	158.0	4	64.6	93.4	
	N	S	2	−66.9		3	−28.8		
	S	S	8	−65.6	158.8	5	−27.9	93.1	
	S	N	7	93.2		6	65.2		
D	W	E	1	37.9	39.9	4	46.0	23.7	
	W	W	2	77.8		3	69.7		
	E	W	8	78.0	40.2	5	69.9	24.1	
	E	E	7	37.8		6	45.8		
D				2 u	39.98		2 u	23.78	
H				2 u'	158.40		2 u'	93.25	

Scalings with magnet away.

	L. M. T.	Scaling.	Temp.	Remarks.
	h. m.	mm.	°	
D	11 02	57.7		Beginning of first set.
D	11 19	58.1		End of first set.
H	11 19	21.0	24.6	Beginning of set.
H	11 33	21.6	24.7	End of set.
D	11 33	57.9		Beginning of second set.
D	11 48	57.8		End of second set.

Torsion observations.

Torsion circle.	Scaling.	Diff.
°	mm.	mm.
133	57.4	
		28.1
163	29.3	
		56.0
103	85.3	
		28.2
133	57.1	
	Sum	112.3
	Mean for 30°	28.1

COMPUTATION.

R=1720.1 mm. H=19880γ

	I.	II.
log $2u$	1.6018	1.3762
" H	4.2984	
" $\left(\frac{f}{f-h}\right)$	0.0068	0.7686
colog $2R$	6.4634	
" $2u'$	7.8002	8.0304
log ε	0.1706	0.1752
ε	1.481	1.497
Mean	1.49	

The scale value of the vertical intensity variometer is determined in a similar manner, but the deflector is placed differently. When deflecting the D magnet the deflector is placed to the north or south (magnetic) of the D variometer, with its axis directed magnetically east and west. When deflecting the Z magnet it is placed to the north or south, with its axis vertical. The formula for computing the Z scale value is of the same form as that derived for H, namely:

$$\varepsilon_z=\frac{2u}{2u'}\cdot\frac{H}{2R}\left(\frac{f}{f-h}\right)$$

u' in this case being the number of millimeters which the Z spot is deflected.

Great care must be exercised when placing the deflector in the various positions in order not to jar the variometer, and thus alter the adjustment. The ordinate with deflector away should be read at the beginning and end of the set, to show whether or not a change has taken place during the observations.

At Cheltenham, where two magnetographs are in operation, the scale value of the Adie Z variometer is determined by the method explained above, but that of the Eschenhagen variometer is obtained by comparing the daily range as given by the two variometers.

TEMPERATURE COEFFICIENTS.

When the variation building is well insulated, so that the diurnal variation of temperature inside is limited to a few tenths of a degree Centigrade and the seasonal variation is very gradual, only approximate values of the temperature coefficients of the H and Z variometers are required. They may sometimes be determined from the regular observatory records by selecting periods free from large disturbances during which there was both a rise and fall of temperature. A comparison of the mean of the 24 hourly ordinates for the days of high temperature with the corresponding mean for the days of low temperature shows the effect of change of temperature. The following example will illustrate the method:

Date.	Temperature.	Mean ordinate.
1907.	°	*mm.*
Jan. 21, 22, 29, 30	−6.79	93.55
Jan. 23, 24, 25, 26	−2.96	95.15
Differences	3.83	1.60
		$q=\frac{1.60}{3.83}\times 4.41=1.9\gamma$

4.41γ being the scale value; that is, an increase of 1° in temperature corresponds to an increase in the ordinate of 1.9γ. In using this method great care must be exercised in the selection of the periods in order to eliminate as far as possible changes in ordinate due to other causes than change in temperature, and an adopted value of temperature coefficient should depend on a number of such determinations. Where the insulation is so good that the temperature in the variation building does not change sufficiently to use the above method, the instrument room may be heated and cooled artificially. For successful results the changes of temperature should be so gradual that the temperature of the magnet will be correctly represented by the thermometer readings and a time should be selected when the changes in *H* and *Z* are apt to be small, unless a second magnetograph is in operation and may be used to determine those changes.

In the case of the *H* variometer there is no reason to expect a change of temperature coefficient. With the *Z* variometer, however, the factors making up the temperature coefficint are so heterogeneous that a radical change of adjustment may produce a change of temperature coefficient.

TEMPERATURE.

In order to determine the temperature of the magnets from the record of the thermograph, the thermometers attached to the *H* and *Z* variometers are read morning and afternoon at times which correspond approximately with the daily extremes of temperature. Under ordinary conditions the variations in temperature may be assumed to be the same for the two variometers. The change in thermometer reading between morning and afternoon compared with the change in ordinate of the thermograph curve will serve to determine the scale value of the thermograph.

TIME SCALE.

The recording apparatus is provided with suitable mechanism for making a short break in the base lines at intervals of an hour. The exact time of occurrence of the first and last time breaks on each magnetogram, as well as of one occurring in the morning, is determined by the click made by the mechanism when the shutters are raised and lowered. The times of stopping and starting the drum are also recorded.

Time observations are made often enough to insure a knowledge of the chronometer correction on local mean time to the nearest tenth of a minute.

READING OF ORDINATES.

It is customary to tabulate the hourly ordinates of *D*, *H*, and *Z* and also the maximum and minimum values for each day and from them to compute the corresponding absolute values. Local mean time is used and the hours are counted from midnight to midnight, 0 to 24. For the determination of the base-line values, ordinates must be measured for the times when absolute observations were made. In the case of the vertical intensity, *H* as well as *Z* ordinates must be measured for the time covered by the dip observations, in order that the value of *H* may be obtained for combining with *I* to compute *Z*. The number of ordinates to be read for a base-line determination depends upon the irregularity of the curve at that time. At the Coast and Geodetic Survey observatories the ordinates are measured from the bottom of the base line to the bottom of the curve and at right angles to the base line. It is generally found that the perpendicular at the end of the base line does not pass through the end of the curve, and this fact must be allowed for when determining the time scale. For measuring the ordinates a reading board has been found very useful. A picture and description of it will be found in "Results of Observations at the Coast and Geodetic Survey Magnetic Observatory at Cheltenham, Md., 1901–1904."

PROGRAMME OF WORK.

While the routine of an observatory is affected somewhat by local conditions, the following programme of work to be done will be modified only in minor details:

(1) Enter the variation room in the morning in time to record the 8 o'clock time break. Read the thermometers, wind the driving clock, see that the lamp is burning brightly and that the spots of light are recording properly. Make psychrometric observations.

(2) Compare the timepiece used with the standard chronometer and wind both of them.

(3) Put new sheets of paper on the seismograph, recording the times of stopping and starting the drums. Wind the clocks when necessary.

(4) Make meteorological observations in the open air.

(5) Enter the variation room in the afternoon in time to record the time break at 16 hours. Read the thermometers. Remove the magnetogram, prick holes for measuring shrinkage, date and put on a new sheet of paper, wind the clock, trim the lamp wick and adjust the light. The lamp requires filling every other day. Record the times of stopping and starting the drum.

(6) Record the time break at 17 hours. Read the thermometers. Examine the lamp and spots of light.

The clock will run for more than 24 hours, but a more uniform rate can usually be secured by winding twice a day. When doing any work in the instrument room requiring light, a ruby lamp should be used or else the front of the recording box should be closed.

Absolute observations are made at least once a week, always on the same day of the week. A week's observations comprise two or three sets of declination, two sets of horizontal intensity, and two or three sets of dip. About once a week the magnetograms are developed and the seismograms are fixed.

With a good chronometer time observations four or five times a month will suffice.

Deflections for the determination of H and Z scale values are made at least once a month. In case of a readjustment the scale value should be determined just before the adjustment and again two or three days afterwards.

GENERAL DIRECTIONS.

Magnetograph record.—The magnetograph record should contain a detailed account of whatever happens to any one of the component parts of the magnetograph, so that the computer will have collected in one place all the information needed to properly interpret the results. If the H variometer is adjusted by turning the torsion head, the amount and direction of change should be recorded. If the position of the control magnets is changed, their position with respect to the suspended magnet and each other, both before and after the change, should be recorded. Similar record should be made of any change in the weights or control magnet of the Z variometer. Any change of adjustment which produces a change of base-line value should be noted also on the base-line computation and on the monthly tabulation of hourly values.

Adjustments.—It is important to make a direct determination of the effect of an adjustment, if possible. For that reason it is desirable to have a magnetogram on the drum at the time, so that a comparison may be made of the relative position of the spots of light before and after the adjustment. Deflections for the determination of scale value should be made just before a readjustment and again two or three days later. When readjusting the *H* variometer, the effort should be made to place the control magnets in such a position as to give a constant scale value. If the deflections are symmetrical about the undeflected position of the magnet, it is safe to assume that the desired result has been obtained. The amount of deflection on *H* or *Z* corresponding to a desired scale value can be readily computed from the formula when the amount of deflection of *D* at the same distance is known. (Compare formula on page 111.)

Arrangement of spots.—The spots of light should be so arranged as to secure as complete and distinct a record as possible. The relative position of curve and base line should be such that increasing ordinates correspond to increasing values of the element. Small ordinates are preferable, but a mixture of positive and negative ordinates is apt to give rise to mistakes. It is undesirable to have the curves so near to each other that there will be many crossings. As the usual effect of a magnetic storm is to diminish the horizontal intensity, the *H* variometer should be so adjusted that the reserve spot will come on at the top of the magnetogram when the regular spot goes off at the bottom. To avoid confusion the *D* variometer should be adjusted so that the reserve spot will come on at the bottom.

Amount of light.—It should be borne in mind that during a magnetic disturbance the motion of the magnet is much more rapid than on a quiet day, and consequently a greater volume of light is required to record its motion on the photographic paper.

Reading of ordinates.—All ordinates are to be read and checked at the observatory. When making the second reading special attention should be directed to the elimination of gross errors, as, for example, misreadings of 5 or 10 mm., reading from the wrong base line, or reading base-line ordinates on the wrong day. Base-line ordinates should be read at the same time as the hourly values. The maximum and minimum ordinates should be compared with the hourly values on the same day as a check against misreadings.

Correction for shrinkage will be made when necessary. Declination scalings will be converted at once to minutes (except in scale-value deflections, where they are to be recorded in millimeters) with the aid of a suitable table, giving the limiting ordinates between which a certain correction is required to convert millimeters to minutes. The torsion factor must always be included in computing the *D* scale value.

Magnetograph temperatures will be obtained directly from the photographic record of the *Z* thermograph, if possible, based on the mean of the *H* and *Z* thermometer readings (corrected). If the variation in temperature is so small that the thermograph trace appears as a straight line between two consecutive thermometer readings, the magnetograph temperatures may be obtained by interpolation between the thermometer readings.

The daily means will be computed on all the monthly tabulation sheets, but the hourly means are required for declination only. A day from which some hourly values are missing or on which a change of base-line value occurred will be omitted in taking means. Missing hourly values may be supplied by interpolation when they are only few in number and occur in a period comparatively free from disturbance.

Magnetic character of day.—The magnetic character of the two halves of each day, a. m. and p. m., will be indicated roughly by the figures 0, 1, 2 on the monthly tabulation for each element, and the character of each Greenwich day as a whole will be tabulated on the same scale, and this tabulation (in duplicate) will be forwarded to the Office as soon as possible after the end of the quarter. At the same time there will be transmitted a table giving the times of occurrence and duration of the principal magnetic disturbances occurring during the quarter.

Absolute observations will be made at least once a week. When field instruments are to be compared with the observatory instruments for purposes of standardization and two observers are available, it is preferable to make simultaneous observations, exchanging the positions of the instruments in the middle of the series in case the relation of the two stations is not known. Where this plan can not be carried out, base-line ordinates will be read for the observations with the field instruments, so that allowance may be made for the variation of the earth's magnetism between the observations with the two sets of instruments.

Transmission of records.—The observatory records will be forwarded to the office monthly, as soon as the necessary computations have been completed. To guard against loss of records in transmission, a summary of the results of absolute observations and a copy of the monthly tabulations of variation observations, base-line determinations, etc., must be retained at the observatory. Such a summary is needed by the observer in order that he may exercise proper control over the observatory work.

Table I.

Correction to observed altitude of the sun for refraction and parallax.

Apparent altitude.	Temperature, centigrade.						Apparent altitude.
	−10°	0°	+10°	+20°	+30°	+40°	
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
5	10 33	10 07	9 43	9 21	9 00	8 40	5
6	9 01	8 39	8 19	8 00	7 42	7 25	6
7	7 51	7 32	7 15	6 59	6 44	6 30	7
8	6 55	6 39	6 24	6 10	5 57	5 44	8
9	6 12	5 58	5 44	5 32	5 20	5 08	9
10	5 35	5 22	5 10	4 59	4 48	4 38	10
11	5 05	4 53	4 42	4 32	4 22	4 13	11
12	4 40	4 29	4 19	4 10	4 01	3 52	12
13	4 17	4 07	3 58	3 50	3 42	3 34	13
14	3 59	3 50	3 41	3 33	3 25	3 18	14
15	3 41	3 33	3 25	3 18	3 11	3 05	15
16	3 28	3 21	3 13	3 06	3 00	2 54	16
17	3 14	3 07	3 00	2 54	2 48	2 43	17
18	3 04	2 57	2 50	2 44	2 38	2 33	18
19	2 53	2 47	2 40	2 34	2 29	2 24	19
20	2 43	2 37	2 31	2 26	2 21	2 16	20
21	2 34	2 28	2 23	2 18	2 13	2 09	21
22	2 25	2 20	2 15	2 10	2 06	2 02	22
23	2 18	2 13	2 08	2 04	1 59	1 55	23
24	2 12	2 06	2 02	1 58	1 54	1 50	24
25	2 05	2 00	1 56	1 52	1 48	1 45	25
26	2 00	1 55	1 51	1 47	1 43	1 40	26
27	1 55	1 50	1 46	1 42	1 38	1 35	27
28	1 49	1 44	1 41	1 38	1 34	1 30	28
29	1 45	1 40	1 37	1 34	1 30	1 26	29
30	1 41	1 36	1 33	1 30	1 27	1 23	30
32	1 32	1 28	1 25	1 22	1 19	1 16	32
34	1 25	1 21	1 18	1 15	1 13	1 10	34
36	1 19	1 15	1 13	1 10	1 08	1 05	36
38	1 13	1 09	1 07	1 05	1 03	1 00	38
40	1 07	1 04	1 02	1 00	0 58	0 56	40
42	1 03	1 00	0 58	0 56	0 54	0 53	42
44	0 58	0 55	0 53	0 51	0 49	0 48	44
46	0 54	0 52	0 50	0 48	0 46	0 45	46
48	0 51	0 49	0 47	0 45	0 43	0 42	48
50	0 47	0 45	0 43	0 41	0 40	0 39	50
55	0 38	0 36	0 35	0 34	0 32	0 30	55
60	0 31	0 30	0 29	0 28	0 27	0 25	60
65	0 25	0 24	0 23	0 22	0 21	0 20	65
70	0 19	0 19	0 18	0 17	0 17	0 16	70
75	0 14	0 14	0 13	0 12	0 12	0 11	75
80	0 09	0 08	0 08	0 08	0 07	0 07	80
85	0 04	0 04	0 04	0 04	0 03	0 03	85
90	0 00	0 00	0 00	0 00	0 00	0 00	90

TABLE II.

Correction in azimuth and altitude of the sun for semidiameter.

Altitude correction = Semidiameter.
Azimuth correction = Semidiameter ÷ cos h.

Date.	Altitude correction.	Azimuth correction.						
		$h=10°$	$h=20°$	$h=30°$	$h=40°$	$h=50°$	$h=60°$	$h=70°$
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
Jan. 1	16 18	16 33	17 21	18 49	21 17	25 21	32 36	47 40
Feb. 1	16 16	16 31	17 19	18 47	21 14	25 18	32 32	47 34
Mar. 1	16 10	16 25	17 12	18 40	21 06	25 09	32 20	47 16
Apr. 1	16 02	16 17	17 04	18 31	20 56	24 57	32 04	46 53
May 1	15 54	16 09	16 55	18 22	20 45	24 44	31 48	46 29
June 1	15 48	16 03	16 49	18 15	20 38	24 35	31 36	46 12
July 1	15 46	16 01	16 47	18 12	20 35	24 32	31 32	46 06
Aug. 1	15 47	16 02	16 48	18 13	20 36	24 33	31 34	46 09
Sept. 1	15 53	16 08	16 54	18 20	20 44	24 43	31 46	46 26
Oct. 1	16 01	16 16	17 03	18 30	20 54	24 55	32 02	46 50
Nov. 1	16 09	16 24	17 11	18 39	21 05	25 07	32 18	47 13
Dec. 1	16 15	16 30	17 18	18 46	21 13	25 17	32 30	47 31

TABLE III.

Latitude from circum-meridian altitudes of the sun.

$$m=\frac{2\sin^2 \frac{1}{2} t}{\sin 1''}$$

t	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m
s.	″	″	″	″	″	″	″	″
0	0	2	8	18	31	49	71	96
10	0	3	9	20	34	52	75	101
20	0	3	11	22	37	56	79	106
30	0	4	12	24	40	59	83	110
40	1	5	14	26	43	63	87	115
50	1	7	16	29	46	67	92	120
60	2	8	18	31	49	71	96	126

t	8^m	9^m	10^m	11^m	12^m	13^m	14^m	15^m
s.	″	″	″	″	″	″	″	″
0	126	159	196	238	283	332	385	442
10	131	165	203	245	291	340	394	452
20	136	171	210	252	299	349	403	461
30	142	177	216	260	307	358	413	472
40	147	183	223	267	315	367	422	482
50	153	190	230	275	323	376	432	492
60	159	196	238	283	332	385	442	502

Table IV.

Latitude from circum-meridian altitudes of the sun.

$A = \cos\delta \cos\phi \operatorname{cosec}\zeta$.

ϕ / δ	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	ϕ / δ
S. 23°	1.95	1.89	1.83	1.77	1.72	1.66	1.62	1.57	1.53	1.48	1.44	S. 23°
22	2.03	1.96	1.90	1.84	1.78	1.72	1.67	1.62	1.58	1.53	1.49	22
21	2.12	2.05	1.97	1.91	1.84	1.79	1.73	1.68	1.63	1.58	1.53	21
20	2.22	2.13	2.05	1.98	1.92	1.85	1.79	1.73	1.68	1.63	1.58	20
19	2.32	2.22	2.14	2.06	1.99	1.92	1.86	1.80	1.74	1.68	1.63	19
18	2.42	2.33	2.23	2.15	2.06	2.00	1.93	1.86	1.80	1.74	1.69	18
17	2.54	2.43	2.33	2.24	2.15	2.07	2.00	1.93	1.86	1.80	1.74	17
16	2.67	2.55	2.44	2.34	2.25	2.16	2.08	2.00	1.93	1.87	1.80	16
15	2.81	2.68	2.56	2.45	2.35	2.25	2.16	2.08	2.00	1.93	1.87	15
14	2.97	2.81	2.69	2.56	2.45	2.35	2.25	2.17	2.08	2.01	1.93	14
13	3.14	2.98	2.83	2.69	2.57	2.46	2.35	2.26	2.17	2.08	2.00	13
12	3.33	3.15	2.98	2.83	2.70	2.57	2.46	2.35	2.26	2.17	2.08	12
11	3.55	3.34	3.15	2.99	2.83	2.70	2.57	2.46	2.35	2.25	2.16	11
10	3.79	3.55	3.34	3.16	2.99	2.84	2.70	2.57	2.46	2.35	2.25	10
9	4.07	3.80	3.56	3.35	3.16	2.99	2.83	2.70	2.57	2.45	2.35	9
8	4.39	4.07	3.80	3.56	3.35	3.16	2.99	2.83	2.69	2.56	2.45	8
7	4.76	4.39	4.07	3.80	3.56	3.34	3.15	2.98	2.83	2.69	2.56	7
6	5.19	4.76	4.39	4.07	3.80	3.55	3.34	3.15	2.98	2.81	2.68	6
5	5.71	5.19	4.76	4.39	4.07	3.79	3.55	3.33	3.14	2.97	2.81	5
4	6.35	5.71	5.19	4.75	4.38	4.07	3.78	3.54	3.32	3.13	2.96	4
3	7.15	6.35	5.71	5.18	4.74	4.37	4.05	3.77	3.53	3.31	3.12	3
2	8.17	7.14	6.34	5.70	5.17	4.73	4.36	4.04	3.76	3.52	3.30	2
S. 1	9.53	8.16	7.13	6.33	5.69	5.16	4.72	4.35	4.04	3.75	3.50	S. 1
0		9.51	8.14	7.12	6.31	5.67	5.14	4.70	4.33	4.01	3.73	0
N. 1			9.49	8.12	7.10	6.29	5.65	5.13	4.69	4.31	3.99	N. 1
2				9.47	8.10	7.07	6.27	5.63	5.10	4.66	4.29	2
3					9.44	8.07	7.04	6.24	5.60	5.08	4.64	3
4						9.41	8.04	7.01	6.21	5.57	5.05	4
5							9.36	8.00	6.97	6.18	5.54	5
6								9.31	7.95	6.93	6.14	6
7									9.25	7.90	6.89	7
8										9.19	7.85	8
9											9.13	9
10												10
11	9.36											11
12	8.00	9.31										12
13	6.97	7.95	9.25									13
14	6.18	6.93	7.90	9.19								14
15	5.54	6.14	6.89	7.85	9.13							15
16	5.02	5.51	6.10	6.84	7.79	9.06						16
17	4.58	4.98	5.47	6.05	6.79	7.73	8.98					17
18	4.21	4.55	4.95	5.42	6.00	6.73	7.66	8.90				18
19	3.89	4.18	4.51	4.91	5.38	5.95	6.67	7.59	8.81			19
20	3.62	3.86	4.15	4.48	4.86	5.33	5.90	6.60	7.51	8.72		20
21	3.37	3.59	3.83	4.11	4.43	4.82	5.28	5.84	6.54	7.43	8.63	21
22	3.16	3.35	3.56	3.80	4.07	4.39	4.77	5.22	5.78	6.46	7.35	22
N. 23	2.97	3.13	3.31	3.52	3.76	4.03	4.35	4.72	5.18	5.71	6.39	N. 23

Table IV—Continued.

Latitude from circum-meridian altitudes of the sun.

$A = \cos\delta \cos\phi \operatorname{cosec}\zeta$.

δ \ ϕ	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	ϕ / δ
S. 23°	1.44	1.41	1.37	1.33	1.30	1.27	1.24	1.21	1.18	1.15	1.12	S. 23°
22	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.24	1.21	1.18	1.15	22
21	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.24	1.21	1.18	21
20	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.24	1.20	20
19	1.63	1.58	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.27	1.23	19
18	1.69	1.63	1.59	1.53	1.49	1.45	1.41	1.37	1.33	1.30	1.26	18
17	1.74	1.69	1.64	1.59	1.53	1.49	1.45	1.41	1.37	1.33	1.30	17
16	1.80	1.74	1.69	1.63	1.58	1.53	1.49	1.45	1.41	1.37	1.33	16
15	1.87	1.80	1.74	1.69	1.63	1.58	1.53	1.49	1.44	1.41	1.36	15
14	1.93	1.87	1.80	1.74	1.68	1.63	1.58	1.53	1.48	1.44	1.40	14
13	2.00	1.93	1.86	1.80	1.74	1.68	1.63	1.58	1.53	1.48	1.43	13
12	2.08	2.00	1.93	1.86	1.80	1.73	1.68	1.62	1.57	1.53	1.47	12
11	2.16	2.08	2.00	1.93	1.86	1.79	1.73	1.67	1.62	1.57	1.51	11
10	2.25	2.16	2.07	2.00	1.92	1.85	1.79	1.72	1.66	1.62	1.56	10
9	2.35	2.25	2.15	2.06	1.99	1.92	1.84	1.78	1.72	1.66	1.60	9
8	2.45	2.34	2.24	2.15	2.06	1.98	1.91	1.84	1.77	1.71	1.65	8
7	2.56	2.44	2.33	2.23	2.14	2.05	1.97	1.90	1.83	1.76	1.70	7
6	2.68	2.55	2.43	2.33	2.22	2.13	2.05	1.96	1.89	1.82	1.75	6
5	2.81	2.67	2.54	2.42	2.32	2.22	2.12	2.03	1.95	1.88	1.81	5
4	2.96	2.80	2.66	2.53	2.41	2.30	2.20	2.11	2.02	1.94	1.87	4
3	3.12	2.95	2.79	2.65	2.52	2.40	2.29	2.19	2.10	2.01	1.93	3
2	3.30	3.10	2.94	2.78	2.64	2.51	2.39	2.28	2.18	2.08	2.00	2
S. 1	3.50	3.29	3.09	2.92	2.76	2.62	2.49	2.37	2.26	2.16	2.07	S. 1
0	3.73	3.49	3.27	3.08	2.90	2.75	2.61	2.48	2.36	2.25	2.14	0
N. 1	3.99	3.71	3.47	3.25	3.06	2.89	2.73	2.59	2.46	2.34	2.23	N. 1
2	4.29	3.97	3.69	3.45	3.23	3.04	2.87	2.71	2.57	2.44	2.32	2
3	4.64	4.27	3.95	3.67	3.43	3.22	3.02	2.84	2.69	2.55	2.42	3
4	5.05	4.61	4.24	3.92	3.64	3.40	3.19	2.99	2.82	2.66	2.52	4
5	5.54	5.02	4.58	4.21	3.89	3.62	3.37	3.16	2.97	2.80	2.64	5
6	6.14	5.51	4.98	4.55	4.18	3.86	3.59	3.35	3.13	2.94	2.77	6
7	6.89	6.10	5.47	4.95	4.51	4.15	3.83	3.56	3.31	3.10	2.91	7
8	7.85	6.84	6.05	5.42	4.91	4.48	4.11	3.80	3.52	3.28	3.07	8
9	9.13	7.79	6.79	6.00	5.38	4.86	4.43	4.07	3.76	3.49	3.25	9
10		9.06	7.73	6.73	5.95	5.33	4.82	4.39	4.03	3.72	3.45	10
11			8.98	7.66	6.67	5.90	5.28	4.77	4.35	3.99	3.68	11
12				8.90	7.59	6.60	5.84	5.22	4.72	4.30	3.94	12
13					8.81	7.51	6.54	5.78	5.16	4.66	4.25	13
14						8.72	7.43	6.46	5.71	5.10	4.61	14
15							8.63	7.35	6.39	5.64	5.04	15
16								8.53	7.26	6.31	5.57	16
17									8.42	7.17	6.23	17
18										8.31	7.07	18
19										9.91	8.20	19
20											9.77	20
21	8.63											21
22	7.35	8.53										22
N. 23	6.39	7.26	8.42									N. 23

TABLE IV—Continued.

Latitude from circum-meridian altitudes of the sun.

$A = \cos \delta \cos \phi \operatorname{cosec} \zeta$

δ \ ϕ	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	ϕ / δ
S. 23°	1.12	1.10	1.07	1.05	1.02	1.00	0.98	0.95	0.93	0.91	0.89	S. 23°
22	1.15	1.12	1.10	1.07	1.04	1.02	1.00	0.97	0.95	0.93	0.91	22
21	1.18	1.15	1.12	1.09	1.07	1.04	1.02	0.99	0.97	0.94	0.92	21
20	1.20	1.17	1.14	1.12	1.09	1.06	1.04	1.01	0.99	0.96	0.94	20
19	1.23	1.20	1.17	1.14	1.11	1.08	1.06	1.03	1.01	0.98	0.96	19
18	1.26	1.23	1.20	1.17	1.14	1.11	1.08	1.05	1.03	1.00	0.98	18
17	1.30	1.26	1.23	1.19	1.16	1.13	1.10	1.07	1.05	1.02	0.99	17
16	1.33	1.29	1.26	1.22	1.19	1.16	1.13	1.10	1.07	1.04	1.01	16
15	1.36	1.32	1.29	1.25	1.22	1.18	1.15	1.12	1.09	1.06	1.03	15
14	1.40	1.36	1.32	1.28	1.24	1.21	1.18	1.14	1.11	1.08	1.05	14
13	1.43	1.39	1.35	1.32	1.27	1.24	1.20	1.17	1.14	1.10	1.07	13
12	1.47	1.43	1.38	1.34	1.30	1.27	1.23	1.19	1.16	1.13	1.10	12
11	1.51	1.47	1.42	1.38	1.34	1.30	1.26	1.22	1.18	1.15	1.12	11
10	1.56	1.51	1.46	1.41	1.37	1.33	1.29	1.25	1.21	1.18	1.14	10
9	1.60	1.55	1.50	1.45	1.40	1.36	1.32	1.28	1.24	1.20	1.16	9
8	1.65	1.59	1.54	1.49	1.44	1.39	1.35	1.31	1.27	1.23	1.19	8
7	1.70	1.64	1.58	1.53	1.48	1.43	1.38	1.34	1.30	1.25	1.21	7
6	1.75	1.69	1.63	1.57	1.52	1.46	1.42	1.37	1.33	1.28	1.24	6
5	1.81	1.74	1.68	1.62	1.56	1.50	1.45	1.40	1.36	1.31	1.27	5
4	1.87	1.79	1.73	1.66	1.60	1.55	1.49	1.44	1.39	1.34	1.30	4
3	1.93	1.85	1.78	1.71	1.65	1.59	1.53	1.48	1.42	1.38	1.33	3
2	2.00	1.91	1.84	1.77	1.70	1.63	1.57	1.52	1.46	1.41	1.36	2
S. 1	2.07	1.98	1.90	1.82	1.75	1.68	1.62	1.56	1.50	1.45	1.39	S. 1
0	2.14	2.05	1.96	1.88	1.80	1.73	1.66	1.60	1.54	1.48	1.43	0
N. 1	2.23	2.13	2.03	1.95	1.86	1.79	1.71	1.65	1.58	1.52	1.47	N. 1
2	2.32	2.21	2.11	2.01	1.93	1.84	1.77	1.69	1.63	1.56	1.50	2
3	2.42	2.30	2.19	2.09	1.99	1.91	1.82	1.75	1.68	1.61	1.54	3
4	2.52	2.39	2.27	2.17	2.06	1.97	1.88	1.80	1.73	1.65	1.59	4
5	2.64	2.50	2.37	2.25	2.14	2.04	1.95	1.86	1.78	1.70	1.63	5
6	2.77	2.61	2.47	2.34	2.23	2.12	2.02	1.92	1.84	1.76	1.68	6
7	2.91	2.74	2.59	2.44	2.32	2.20	2.09	1.99	1.90	1.81	1.73	7
8	3.07	2.88	2.71	2.56	2.42	2.29	2.17	2.06	1.97	1.87	1.79	8
9	3.25	3.04	2.85	2.68	2.53	2.39	2.26	2.14	2.04	1.94	1.85	9
10	3.45	3.21	3.00	2.81	2.65	2.49	2.36	2.23	2.11	2.01	1.91	10
11	3.68	3.41	3.17	2.96	2.78	2.61	2.46	2.32	2.20	2.08	1.98	11
12	3.94	3.63	3.37	3.13	2.93	2.74	2.58	2.43	2.29	2.17	2.05	12
13	4.25	3.90	3.59	3.32	3.09	2.89	2.70	2.54	2.39	2.25	2.13	13
14	4.61	4.19	3.84	3.54	3.28	3.05	2.85	2.66	2.50	2.35	2.22	14
15	5.04	4.55	4.14	3.79	3.49	3.23	3.00	2.80	2.62	2.46	2.31	15
16	5.57	4.98	4.49	4.08	3.74	3.44	3.18	2.96	2.76	2.58	2.42	16
17	6.23	5.49	4.91	4.43	4.02	3.68	3.39	3.13	2.91	2.71	2.54	17
18	7.07	6.14	5.42	4.84	4.36	3.96	3.62	3.33	3.08	2.86	2.66	18
19	8.20	6.97	6.05	5.34	4.76	4.29	3.90	3.56	3.28	3.03	2.81	19
20	9.77	8.08	6.87	5.96	5.25	4.69	4.22	3.83	3.50	3.22	2.97	20
21		9.63	7.96	6.76	5.87	5.17	4.61	4.15	3.77	3.44	3.16	21
22			9.48	7.83	6.65	5.77	5.08	4.53	4.07	3.70	3.38	22
N. 23				9.32	7.71	6.54	5.67	4.99	4.45	4.00	3.63	N. 23

TABLE IV—Continued.

Latitude from circum-meridian altitudes of the sun.

$A = \cos\delta \cos\phi \operatorname{cosec}\zeta$

$\delta \backslash \phi$	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	ϕ / δ
S. 23°	0.89	0.87	0.85	0.83	0.81	0.79	0.77	0.75	0.74	0.72	0.70	S. 23°
22	0.91	0.88	0.86	0.84	0.82	0.80	0.78	0.77	0.75	0.73	0.71	22
21	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	21
20	0.94	0.92	0.90	0.87	0.85	0.83	0.81	0.79	0.77	0.75	0.73	20
19	0.96	0.93	0.91	0.89	0.87	0.84	0.82	0.80	0.78	0.76	0.74	19
18	0.98	0.95	0.93	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	18
17	0.99	0.97	0.94	0.92	0.90	0.87	0.85	0.83	0.81	0.79	0.77	17
16	1.01	0.99	0.96	0.94	0.91	0.89	0.86	0.84	0.82	0.80	0.78	16
15	1.03	1.01	0.98	0.95	0.93	0.90	0.88	0.86	0.83	0.81	0.79	15
14	1.05	1.03	1.00	0.97	0.94	0.92	0.89	0.87	0.85	0.82	0.80	14
13	1.07	1.05	1.02	0.99	0.96	0.93	0.91	0.88	0.86	0.84	0.81	13
12	1.10	1.07	1.04	1.01	0.98	0.95	0.92	0.90	0.87	0.85	0.82	12
11	1.12	1.09	1.06	1.02	1.00	0.97	0.94	0.91	0.89	0.86	0.84	11
10	1.14	1.11	1.08	1.04	1.01	0.98	0.96	0.93	0.90	0.88	0.85	10
9	1.16	1.13	1.10	1.06	1.03	1.00	0.97	0.94	0.92	0.89	0.86	9
8	1.19	1.15	1.12	1.08	1.05	1.02	0.99	0.96	0.93	0.90	0.88	8
7	1.21	1.18	1.14	1.11	1.07	1.04	1.01	0.98	0.95	0.92	0.89	7
6	1.24	1.20	1.17	1.13	1.09	1.06	1.03	0.99	0.96	0.93	0.91	6
5	1.27	1.23	1.19	1.15	1.11	1.08	1.04	1.01	0.98	0.95	0.92	5
4	1.30	1.26	1.21	1.17	1.14	1.10	1.07	1.03	1.00	0.97	0.94	4
3	1.33	1.28	1.24	1.20	1.16	1.12	1.09	1.05	1.02	0.98	0.95	3
2	1.36	1.31	1.27	1.23	1.18	1.14	1.11	1.07	1.03	1.00	0.97	2
S. 1	1.39	1.34	1.30	1.25	1.21	1.17	1.13	1.09	1.05	1.02	0.98	S. 1
0	1.43	1.38	1.33	1.28	1.24	1.19	1.15	1.11	1.07	1.04	1.00	0
N. 1	1.47	1.41	1.36	1.31	1.26	1.22	1.17	1.13	1.09	1.05	1.02	N. 1
2	1.50	1.45	1.39	1.34	1.29	1.24	1.20	1.16	1.11	1.07	1.04	2
3	1.54	1.48	1.43	1.37	1.32	1.27	1.22	1.18	1.14	1.09	1.06	3
4	1.59	1.52	1.46	1.41	1.35	1.30	1.25	1.20	1.16	1.12	1.07	4
5	1.63	1.56	1.50	1.44	1.38	1.33	1.28	1.23	1.18	1.14	1.10	5
6	1.68	1.61	1.54	1.48	1.42	1.36	1.31	1.26	1.21	1.16	1.12	6
7	1.73	1.66	1.58	1.52	1.46	1.40	1.34	1.29	1.23	1.19	1.14	7
8	1.79	1.71	1.63	1.56	1.49	1.43	1.37	1.32	1.26	1.21	1.16	8
9	1.85	1.76	1.68	1.60	1.54	1.47	1.41	1.35	1.29	1.24	1.19	9
10	1.91	1.82	1.73	1.65	1.58	1.51	1.44	1.38	1.32	1.27	1.21	10
11	1.98	1.88	1.79	1.70	1.63	1.55	1.48	1.42	1.35	1.30	1.24	11
12	2.05	1.95	1.85	1.76	1.67	1.60	1.52	1.45	1.39	1.33	1.27	12
13	2.13	2.02	1.91	1.82	1.73	1.64	1.57	1.49	1.43	1.36	1.30	13
14	2.22	2.10	1.98	1.88	1.78	1.70	1.61	1.54	1.46	1.40	1.33	14
15	2.31	2.18	2.06	1.95	1.85	1.75	1.66	1.58	1.51	1.43	1.37	15
16	2.42	2.27	2.14	2.02	1.91	1.81	1.72	1.63	1.55	1.47	1.40	16
17	2.54	2.38	2.23	2.10	1.98	1.88	1.77	1.68	1.60	1.51	1.44	17
18	2.66	2.49	2.33	2.19	2.06	1.94	1.84	1.74	1.65	1.56	1.48	18
19	2.81	2.62	2.44	2.29	2.15	2.02	1.90	1.80	1.70	1.61	1.52	19
20	2.97	2.76	2.57	2.40	2.24	2.10	1.98	1.86	1.76	1.66	1.57	20
21	3.16	2.92	2.70	2.52	2.35	2.20	2.06	1.94	1.82	1.72	1.62	21
22	3.38	3.10	2.86	2.65	2.46	2.30	2.15	2.01	1.89	1.78	1.68	22
N. 23	3.63	3.31	3.04	2.80	2.60	2.41	2.25	2.10	1.97	1.85	1.74	N. 23

Table IV—Continued.

Latitude from circum-meridian altitudes of the sun.

$A = \cos \delta \cos \phi \operatorname{cosec} \zeta$

ϕ / δ	45°	46°	47°	48°	49°	50°	55°	60°	65°	70°	ϕ / δ
S. 23°	0.70	0.69	0.67	0.65	0.64	0.62	0.54	0.46	0.39		S. 23°
22	0.71	0.69	0.68	0.66	0.64	0.63	0.55	0.47	0.39		22
21	0.72	0.71	0.69	0.67	0.65	0.63	0.55	0.47	0.40		21
20	0.73	0.71	0.70	0.68	0.66	0.64	0.56	0.48	0.40		20
19	0.74	0.72	0.71	0.69	0.67	0.65	0.56	0.48	0.40	0.32	19
18	0.76	0.74	0.72	0.70	0.68	0.66	0.57	0.49	0.40	0.33	18
17	0.77	0.75	0.73	0.71	0.69	0.67	0.58	0.49	0.41	0.33	17
16	0.78	0.76	0.74	0.72	0.70	0.68	0.58	0.50	0.41	0.33	16
15	0.79	0.77	0.75	0.73	0.70	0.69	0.59	0.50	0.41	0.33	15
14	0.80	0.78	0.76	0.74	0.71	0.69	0.60	0.50	0.42	0.33	14
13	0.81	0.79	0.77	0.75	0.72	0.70	0.60	0.51	0.42	0.34	13
12	0.82	0.80	0.78	0.76	0.73	0.71	0.61	0.51	0.42	0.34	12
11	0.84	0.81	0.79	0.77	0.74	0.72	0.62	0.52	0.43	0.34	11
10	0.85	0.82	0.80	0.78	0.75	0.73	0.62	0.52	0.43	0.34	10
9	0.86	0.84	0.81	0.79	0.76	0.74	0.63	0.53	0.43	0.34	9
8	0.88	0.85	0.82	0.80	0.78	0.75	0.64	0.53	0.44	0.35	8
7	0.89	0.86	0.84	0.81	0.78	0.76	0.64	0.54	0.44	0.35	7
6	0.91	0.88	0.85	0.82	0.80	0.77	0.65	0.54	0.44	0.35	6
5	0.92	0.89	0.86	0.83	0.81	0.78	0.66	0.55	0.45	0.35	5
4	0.94	0.90	0.88	0.85	0.82	0.79	0.67	0.55	0.45	0.35	4
3	0.95	0.92	0.89	0.86	0.83	0.80	0.68	0.56	0.46	0.36	3
2	0.97	0.93	0.90	0.87	0.84	0.82	0.68	0.57	0.46	0.36	2
S. 1	0.98	0.95	0.92	0.89	0.86	0.83	0.69	0.57	0.46	0.36	S. 1
0	1.00	0.97	0.93	0.90	0.87	0.84	0.70	0.58	0.47	0.36	0
N. 1	1.02	0.98	0.95	0.92	0.88	0.85	0.71	0.58	0.47	0.37	N. 1
2	1.04	1.00	0.96	0.93	0.90	0.86	0.72	0.59	0.47	0.37	2
3	1.06	1.02	0.98	0.94	0.91	0.88	0.73	0.60	0.48	0.37	3
4	1.07	1.04	0.99	0.96	0.93	0.89	0.74	0.60	0.48	0.37	4
5	1.10	1.05	1.02	0.98	0.94	0.91	0.75	0.61	0.48	0.38	5
6	1.12	1.07	1.03	0.99	0.96	0.92	0.76	0.61	0.49	0.38	6
7	1.14	1.10	1.05	1.01	0.97	0.94	0.77	0.62	0.49	0.38	7
8	1.16	1.12	1.07	1.03	0.99	0.95	0.78	0.63	0.50	0.38	8
9	1.19	1.14	1.09	1.05	1.01	0.97	0.79	0.64	0.50	0.39	9
10	1.21	1.16	1.12	1.07	1.03	0.98	0.80	0.64	0.51	0.39	10
11	1.24	1.19	1.14	1.09	1.05	1.00	0.81	0.65	0.51	0.39	11
12	1.27	1.22	1.16	1.11	1.07	1.02	0.82	0.66	0.52	0.39	12
13	1.30	1.24	1.19	1.14	1.09	1.04	0.84	0.67	0.52	0.40	13
14	1.33	1.27	1.22	1.16	1.11	1.06	0.85	0.67	0.53	0.40	14
15	1.37	1.30	1.24	1.19	1.13	1.08	0.86	0.68	0.53	0.40	15
16	1.40	1.34	1.27	1.21	1.16	1.10	0.88	0.69	0.54	0.41	16
17	1.44	1.37	1.30	1.24	1.18	1.13	0.89	0.70	0.54	0.41	17
18	1.48	1.41	1.34	1.27	1.21	1.15	0.91	0.71	0.55	0.41	18
19	1.52	1.45	1.37	1.31	1.24	1.18	0.92	0.72	0.56	0.42	19
20	1.57	1.49	1.41	1.34	1.27	1.21	0.94	0.73	0.56	0.42	20
21	1.62	1.53	1.45	1.38	1.30	1.24	0.96	0.74	0.57	0.46	21
22	1.68	1.58	1.50	1.41	1.34	1.27	0.98	0.75	0.57	0.50	22
N. 23	1.74	1.64	1.54	1.46	1.38	1.30	1.00	0.76	0.58	0.54	N. 23

TABLE V.

Correction for rate of chronometer (oscillations).

(The correction is to be added when chronometer was losing and subtracted when gaining.)

Daily rate.	Time of one oscillation.						
	$3^s.0$	$4^s.0$	$5^s.0$	$6^s.0$	$7^s.0$	$8^s.0$	$9^s.0$
s.	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
1.0	0.00003	0.00005	0.00006	0.00007	0.00008	0.00009	0.00010
2.0	07	09	12	14	16	19	21
3.0	10	14	17	21	24	28	31
4.0	14	19	23	28	32	37	42
5.0	17	23	29	35	41	46	52
6.0	21	28	35	42	49	56	63
7.0	24	32	40	49	57	65	73
8.0	28	37	46	56	65	74	83
9.0	31	42	52	62	73	83	94
10.0	35	46	58	69	81	93	104
20.0	69	93	116	139	162	185	208
30.0	0.00104	0.00139	0.00174	0.00208	0.00243	0.00278	0.00313
$3^m56^s.6$	0.00821	0.01095	0.01369	0.01643	0.01917	0.02190	0.02464

The last line is added for the case in which a sidereal chronometer was used in observing.

TABLE VI.

Torsion factor (oscillations).

(Values of [log 5400–log (5400–h)] are given in units of the fifth decimal place; h is expressed in minutes of arc.)

h	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0	1	2	2	3	4	5	6	6	7
1	8	9	10	10	11	12	13	14	14	15
2	16	17	18	18	19	20	21	22	23	23
3	24	25	26	27	27	28	29	30	31	31
4	32	33	34	35	35	36	37	38	39	39
5	40	41	42	43	43	44	45	46	47	47
6	48	49	50	51	51	52	53	54	55	55
7	56	57	58	59	59	60	61	62	63	63
8	64	65	66	67	68	68	69	70	71	72
9	72	73	74	75	76	76	77	78	79	80

Table VII.

Reduction of log C from 20° C. to other temperatures (deflections).

(The corrections are given in terms of the fifth decimal place in the logarithm.)

Temp.	Corr'n.	Temp.	Corr'n.	Temp.	Corr'n.	Temp.	Corr'n.
°		°		°		°	
0	+50	10	+25	20	0	30	−25
1	47.5	11	22.5	21	− 2.5	31	27.5
2	45	12	20	22	5	32	30.
3	42.5	13	17.5	23	7.5	33	32.5
4	40	14	15	24	10	34	35
5	37.5	15	12.5	25	12.5	35	37.5
6	35	16	10	26	15	36	40
7	32.5	17	7.5	27	17.5	37	42.5
8	30	18	5	28	20	38	45
9	27.5	19	+ 2.5	29	22.5	39	47.5
10	+25	20	0	30	−25	40	−50

Table VIII.

Correction for lack of balance of dip needle.

(dI is the difference of the two values of dip from observations before and after reversal of polarities. The correction is always to be added.)

I	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°	75°
dI											
° ′	′	′	′	′	′	′	′	′	′	′	′
0 10	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.03	0.03
20	0.00	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.06	0.08	0.11
30	0.00	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.14	0.18	0.24
40	0.01	0.05	0.06	0.09	0.11	0.14	0.16	0.20	0.25	0.32	0.43
50	0.02	0.08	0.10	0.15	0.18	0.22	0.26	0.31	0.40	0.50	0.67
1 00	0.03	0.10	0.15	0.22	0.27	0.32	0.38	0.45	0.56	0.72	0.97
10	0.05	0.13	0.20	0.30	0.36	0.43	0.51	0.62	0.76	0.98	1.33
20	0.07	0.17	0.26	0.39	0.47	0.56	0.67	0.81	0.99	1.28	1.74
30	0.09	0.22	0.33	0.49	0.59	0.70	0.85	1.03	1.26	1.62	2.20
40	0.12	0.27	0.41	0.60	0.73	0.86	1.05	1.27	1.56	2.00	2.71
50	0.15	0.33	0.50	0.73	0.88	1.04	1.26	1.54	1.89	2.42	3.28
2 00	0.17	0.39	0.60	0.87	1.05	1.24	1.50	1.82	2.25	2.88	3.91

TABLE IX.

Diurnal variation of declination.

Month.	Jan., Feb., Nov., Dec.				Mar., Apr., Sept., Oct.				May, June, July, Aug.			
Hour.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.
	′	′	′	′	′	′	′	′	′	′	′	′
1	−0.3	−0.2	−0.2	−0.2	−0.2	+0.2	−0.2	−0.2	−0.8	+0.1	−0.1	−0.2
2	−0.1	−0.2	−0.1	−0.3	−0.1	+0.3	−0.1	−0.2	−0.6	+0.2	0.0	−0.1
3	0.0	−0.1	−0.1	−0.3	+0.2	+0.6	0.0	−0.1	−0.2	+0.4	+0.2	0.0
4	+0.1	+0.1	0.0	−0.2	+0.4	+0.8	+0.2	+0.1	+0.9	+0.8	+0.4	+0.2
5	+0.3	+0.3	+0.1	−0.1	+1.1	+1.1	+0.4	+0.3	+2.6	+1.8	+0.7	+0.6
6	+0.5	+0.6	0.0	0.0	+2.1	+1.9	+0.9	+0.8	+4.5	+3.5	+1.9	+1.8
7	+1.0	+1.1	0.0	+0.1	+3.4	+3.3	+2.2	+2.0	+6.2	+5.0	+3.6	+3.4
8	+1.7	+2.1	+1.0	+1.1	+4.7	+4.2	+3.1	+2.6	+7.2	+5.3	+3.5	+3.6
9	+2.2	+2.9	+1.9	+2.3	+4.7	+3.6	+2.7	+2.5	+6.8	+3.9	+2.1	+2.5
10	+1.8	+2.3	+2.0	+2.8	+3.6	+1.7	+1.3	+1.8	+4.4	+0.9	+0.2	+1.0
11	+1.0	+0.5	+0.9	+2.1	+1.7	−1.0	−0.4	+0.6	+0.8	−2.2	−1.5	−0.4
12	+0.1	−1.6	−0.5	+0.5	−0.5	−3.2	−1.7	−0.6	−2.0	−4.3	−2.5	−1.3
13	−0.7	−2.8	−1.4	−0.8	−2.3	−4.2	−2.1	−1.5	−3.8	−5.0	−2.5	−1.9
14	−1.5	−2.8	−1.8	−1.5	−3.0	−4.0	−1.9	−2.0	−5.0	−4.6	−2.0	−2.1
15	−1.6	−2.1	−1.5	−1.7	−3.3	−3.0	−1.4	−1.9	−5.3	−3.4	−1.3	−1.9
16	−1.5	−1.4	−0.9	−1.4	−3.1	−1.7	−0.8	−1.4	−4.6	−2.0	−0.7	−1.4
17	−1.3	−0.6	0.0	−1.0	−2.6	−0.7	−0.5	−0.9	−3.5	−0.5	−0.4	−0.7
18	−0.8	−0.1	+0.1	−0.6	−2.0	−0.4	−0.4	−0.6	−2.2	+0.1	−0.4	−0.5
19	−0.4	+0.2	+0.1	−0.3	−1.4	−0.2	−0.3	−0.5	−1.1	+0.1	−0.3	−0.5
20	−0.2	+0.4	+0.2	−0.1	−0.9	0.0	−0.2	−0.4	−0.8	−0.1	−0.3	−0.5
21	−0.2	+0.6	+0.1	0.0	−0.8	+0.1	−0.2	−0.3	−0.7	0.0	−0.3	−0.5
22	−0.2	+0.4	0.0	0.0	−0.7	+0.2	−0.3	−0.2	−0.9	0.0	−0.2	−0.4
23	−0.2	+0.3	0.0	−0.1	−0.7	+0.2	−0.3	−0.2	−0.9	+0.1	−0.2	−0.3
24	−0.2	+0.1	−0.1	−0.2	−0.6	+0.2	−0.2	−0.2	−0.9	+0.1	−0.2	−0.2

		° ′
Sitka, Alaska, 1902–1906.	Mean declination,	29 56.6 E.
Cheltenham, Md., 1902–1906.		5 13.9 W.
Honolulu, Hawaii, 1902–1906.		9 20.9 E.
Vieques, Porto Rico, 1904–1906.		1 38.4 W.

A plus sign indicates that east declination is greater or west declination is less than the mean for the day.

Table X.

Diurnal variation of dip.

Month.	Jan., Feb., Nov., Dec.				Mar., Apr., Sept., Oct.				May, June, July, Aug.			
Hour.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.
	′	′	′	′	′	′	′	′	′	′	′	′
1	−0.2	−0.1	+0.6	+0.2	−0.4	−0.3	+0.6	+0.3	−0.4	−0.1	+0.4	+0.2
2	−0.2	−0.2	+0.6	+0.2	−0.5	−0.3	+0.6	+0.4	−0.4	−0.1	+0.4	+0.2
3	−0.2	−0.2	+0.6	+0.2	−0.6	−0.3	+0.6	+0.2	−0.4	−0.1	+0.5	+0.2
4	−0.2	−0.2	+0.4	+0.2	−0.6	−0.3	+0.5	+0.2	−0.4	0.0	+0.5	+0.2
5	−0.2	−0.3	+0.4	0.0	−0.6	−0.4	+0.5	+0.2	−0.5	−0.1	+0.6	+0.2
6	−0.3	−0.3	+0.2	0.0	−0.5	−0.3	+0.6	+0.2	−0.6	−0.1	+0.8	+0.2
7	−0.2	−0.2	0.0	−0.2	−0.4	0.0	+1.0	−0.1	−0.4	+0.2	+0.9	−0.2
8	−0.2	−0.1	0.0	−0.5	−0.2	+0.5	+1.1	−0.4	−0.2	+0.8	+0.4	−0.3
9	−0.2	+0.2	0.0	−0.6	+0.2	+1.0	+0.6	−0.6	+0.2	+1.3	−0.3	−0.6
10	+0.1	+0.6	−0.4	−0.7	+0.6	+1.2	−0.4	−0.8	+0.8	+1.3	−1.0	−0.7
11	+0.4	+1.0	−1.0	−0.7	+1.0	+1.1	−1.4	−0.9	+1.0	+0.8	−1.4	−0.8
12	+0.7	+1.0	−1.4	−0.6	+1.2	+0.7	−1.8	−0.8	+1.2	+0.2	−1.5	−0.8
13	+0.8	+0.7	−1.4	−0.2	+1.2	+0.3	−1.8	−0.6	+1.1	−0.4	−1.3	−0.5
14	+0.8	+0.4	−1.2	0.0	+1.0	0.0	−1.4	−0.4	+0.8	−0.7	−0.9	−0.1
15	+0.5	0.0	−0.7	+0.2	+0.6	−0.3	−0.9	0.0	+0.4	−0.7	−0.6	+0.2
16	+0.2	−0.2	−0.2	+0.4	+0.3	−0.3	−0.4	+0.2	+0.1	−0.6	−0.3	+0.4
17	−0.1	−0.3	+0.2	+0.4	0.0	−0.3	0.0	+0.4	−0.2	−0.3	+0.1	+0.5
18	−0.2	−0.3	+0.4	+0.3	−0.2	−0.3	+0.2	+0.4	−0.4	−0.2	+0.2	+0.4
19	−0.2	−0.2	+0.4	+0.3	−0.3	−0.3	+0.2	+0.4	−0.3	−0.2	+0.3	+0.4
20	−0.2	−0.2	+0.4	+0.2	−0.4	−0.3	+0.4	+0.4	−0.3	−0.2	+0.4	+0.3
21	−0.2	−0.2	+0.5	+0.2	−0.4	−0.3	+0.4	+0.4	−0.3	−0.2	+0.4	+0.2
22	−0.2	−0.2	+0.6	+0.2	−0.4	−0.3	+0.4	+0.3	−0.2	−0.2	+0.4	+0.2
23	−0.2	−0.2	+0.6	+0.2	−0.4	−0.3	+0.4	+0.2	−0.4	−0.2	+0.4	+0.2
24	−0.2	−0.2	+0.6	+0.2	−0.4	−0.3	+0.4	+0.2	−0.4	−0.2	+0.4	+0.2

		°	′
Sitka, Alaska, 1905–6.	Mean dip,	74	42.1
Cheltenham, Md., 1902–6.		70	24.2
Honolulu, Hawaii, 1905–6.		40	03.9
Vieques, Porto Rico, 1905–6.		49	19.6

A plus sign indicates a value greater than the mean for the day.

TABLE XI.

Diurnal variation of horizontal intensity.

Month.	Jan., Feb., Nov., Dec.				Mar., Apr., Sept., Oct.				May, June, July, Aug.			
Hour.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.	Sitka.	Ch.	Hon.	P. R.
	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ	γ
1	+ 1	+ 3	− 5	−3	+ 6	+ 5	− 5	− 3	+ 7	+ 3	− 5	− 4
2	+ 2	+ 4	− 5	−2	+ 7	+ 5	− 4	− 2	+ 7	+ 2	− 4	− 3
3	+ 2	+ 4	− 4	−1	+ 8	+ 6	− 4	− 2	+ 7	+ 2	− 4	− 3
4	+ 3	+ 5	− 3	0	+ 8	+ 6	− 3	0	+ 8	+ 2	− 4	− 2
5	+ 3	+ 6	− 3	+1	+ 8	+ 6	− 2	0	+ 9	+ 3	− 4	− 2
6	+ 3	+ 6	− 1	+3	+ 7	+ 6	− 2	+ 1	+ 9	+ 4	− 3	0
7	+ 3	+ 5	+ 3	+7	+ 6	0	− 3	+ 2	+ 7	− 2	− 1	0
8	+ 3	+ 1	+ 5	+9	+ 2	− 9	− 4	+ 2	+ 1	−14	+ 3	+ 2
9	0	− 5	+ 6	+8	− 5	−19	− 2	+ 5	− 7	−24	+ 5	+ 7
10	− 4	−13	+ 8	+6	−11	−24	+ 3	+ 8	−17	−26	+ 8	+12
11	− 8	−21	+11	+5	−17	−23	+ 8	+10	−21	−19	+ 9	+14
12	−11	−20	+10	+2	−19	−16	+12	+10	−22	− 8	+10	+12
13	−12	−14	+ 8	−2	−18	− 8	+13	+ 8	−19	+ 4	+10	+ 8
14	− 9	− 6	+ 5	−3	−13	0	+11	+ 8	−13	+12	+ 9	+ 4
15	− 5	+ 1	+ 2	−4	− 8	+ 5	+ 7	− 1	− 5	+14	+ 5	− 1
16	− 1	+ 6	0	−4	− 3	+ 8	+ 2	− 4	+ 1	+13	+ 2	− 6
17	+ 3	+ 6	− 3	−4	+ 1	+ 6	− 2	− 5	+ 5	+ 8	− 2	− 8
18	+ 5	+ 6	− 4	−4	+ 5	+ 7	− 3	− 5	+ 7	+ 4	− 4	− 7
19	+ 5	+ 5	− 5	−4	+ 6	+ 7	− 3	− 4	+ 6	+ 4	− 5	− 6
20	+ 4	+ 5	− 5	−3	+ 6	+ 6	− 4	− 4	+ 5	+ 4	− 5	− 5
21	+ 4	+ 4	− 6	−3	+ 6	+ 6	− 5	− 4	+ 5	+ 4	− 5	− 5
22	+ 3	+ 3	− 6	−2	+ 6	+ 6	− 5	− 4	+ 6	+ 4	− 5	− 4
23	+ 2	+ 4	− 5	−2	+ 6	+ 6	− 4	− 3	+ 6	+ 4	− 4	− 4
24	+ 3	+ 4	− 5	−1	+ 7	+ 6	− 4	− 2	+ 7	+ 4	− 4	− 3

Sitka, Alaska, 1902–6. Mean horizontal intensity,	γ 15491
Cheltenham, Md., 1902–6.	20121
Honolulu, Hawaii, 1902–6.	29245
Vieques, Porto Rico, 1903–6.	29278

A plus sign indicates a value greater than the mean for the day.

Table XII.

Multiples of the sines of the angles 22°.5, 45°, and 67°.5.

	22°.5	45°	67°.5		22°.5	45°	67°.5
1	0.38	0.71	0.92	51	19.52	36.06	47.12
2	0.77	1.41	1.85	52	19.90	36.77	48.04
3	1.15	2.12	2.77	53	20.28	37.48	48.97
4	1.53	2.83	3.70	54	20.66	38.18	49.89
5	1.91	3.54	4.62	55	21.05	38.89	50.81
6	2.30	4.24	5.54	56	21.43	39.60	51.74
7	2.68	4.95	6.47	57	21.81	40.31	52.66
8	3.06	5.66	7.39	58	22.20	41.01	53.59
9	3.44	6.36	8.31	59	22.58	41.72	54.51
10	3.83	7.07	9.24	60	22.96	42.43	55.43
11	4.21	7.78	10.16	61	23.34	43.13	56.36
12	4.59	8.49	11.09	62	23.73	43.84	57.28
13	4.97	9.19	12.01	63	24.11	44.55	58.20
14	5.36	9.90	12.93	64	24.49	45.25	59.13
15	5.74	10.61	13.86	65	24.87	45.96	60.05
16	6.12	11.31	14.78	66	25.26	46.67	60.98
17	6.51	12.02	15.71	67	25.64	47.38	61.90
18	6.89	12.73	16.63	68	26.02	48.08	62.82
19	7.27	13.44	17.55	69	26.41	48.79	63.75
20	7.65	14.14	18.48	70	26.79	49.50	64.67
21	8.04	14.85	19.40	71	27.17	50.20	65.60
22	8.42	15.56	20.33	72	27.55	50.91	66.52
23	8.80	16.26	21.25	73	27.94	51.62	67.44
24	9.18	16.97	22.17	74	28.32	52.33	68.37
25	9.57	17.68	23.10	75	28.70	53.03	69.29
26	9.95	18.38	24.02	76	29.08	53.74	70.21
27	10.33	19.09	24.94	77	29.47	54.45	71.14
28	10.72	19.80	25.87	78	29.85	55.15	72.06
29	11.10	20.51	26.79	79	30.23	55.86	72.99
30	11.48	21.21	27.72	80	30.61	56.57	73.91
31	11.86	21.92	28.64	81	31.00	57.28	74.83
32	12.25	22.63	29.56	82	31.38	57.98	75.76
33	12.63	23.33	30.49	83	31.76	58.69	76.68
34	13.01	24.04	31.41	84	32.15	59.40	77.61
35	13.39	24.75	32.34	85	32.53	60.10	78.53
36	13.78	25.46	33.26	86	32.91	60.81	79.45
37	14.16	26.16	34.18	87	33.29	61.52	80.38
38	14.54	26.87	35.11	88	33.68	62.23	81.30
39	14.92	27.58	36.03	89	34.06	62.93	82.23
40	15.31	28.28	36.96	90	34.44	63.64	83.15
41	15.69	28.99	37.88	91	34.82	64.35	84.07
42	16.07	29.70	38.80	92	35.21	65.05	85.00
43	16.46	30.41	39.73	93	35.59	65.76	85.92
44	16.84	31.11	40.65	94	35.97	66.47	86.84
45	17.22	31.82	41.57	95	36.35	67.18	87.77
46	17.60	32.53	42.50	96	36.74	67.88	88.69
47	17.99	33.23	43.42	97	37.12	68.59	89.62
48	18.37	33.94	44.35	98	37.50	69.30	90.54
49	18.75	34.65	45.27	99	37.89	70.00	91.46
50	19.13	35.36	46.19	100	38.27	70.71	92.39

Proportional parts.

22°.5

0.1	.04
.2	.08
.3	.11
.4	.15
.5	.19
.6	.23
.7	.27
.8	.31
.9	.34

45°

0.1	.07
.2	.14
.3	.21
.4	.28
.5	.35
.6	.42
.7	.49
.8	.57
.9	.64

67°.5

0.1	.09
.2	.18
.3	.28
.4	.37
.5	.46
.6	.55
.7	.65
.8	.74
.9	.83